FUNCTIONALITY, COMPLEXITY, AND APPROACHES TO ASSESSMENT OF RESILIENCE UNDER CONSTRAINED ENERGY AND INFORMATION

DISSERTATION

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Abstract

This research, sponsored by the Department of Defense Systems Engineering Research Center, developed a methodology to measure the functionality and complexity of engineered systems in order to assess system resilience. While system functions, functionality, and complexity are widely used concepts in systems engineering, there is signficant diversity in their definitions and no unified approach to measurement. This research establishes a method for measuring impacts to functionality in dynamic engineered systems based on changes in kinetic energy. This metric is applied at particular levels of abstraction and system scales, consistent with the established multiscale nature of systems. By measuring system behavior in context with expected scenarios, it is possible to estimate expected functionality or set bounds on a system's maximum functionality.

Functionality and system effectiveness is heavily influenced by the amount of available energy and the information a system has about its environment. A framework is needed for quickly assessing the impact of changes in information in order to drive system architecture and design. This research relates functionality to the information content required to describe a system using principles from information theory and complexity theory.

The theory developed in this research is validated using an aircraft simulation with 2 degrees of freedom, which is used to generate data against a large number of scenarios for several system instantiations. Analysis of the results of this simulation shows consistency with the developed theory and establishes a solid basis for further exploration and application of this research.

Table of Contents

Page

Page

List of Figures

Figure Page

List of Tables

List of Abbreviations

Abbreviation Page

FUNCTIONALITY, COMPLEXITY, AND APPROACHES TO ASSESSMENT OF RESILIENCE UNDER CONSTRAINED ENERGY AND INFORMATION

I. Introduction

Over the past century, engineered systems have increased dramatically in what is known colloquially as "complexity". Complexity creeps in as desire grows for increased automation or solutions to daunting challenges. Its characteristics appear in suddenly critical problems of modern civilization that were nonexistent mere years before. Complexity is intimately tied to system behavior – the performance of functions – although the nature of the relationship appears murky through the lens of our current systems engineering tools.

Engineers lament the multitude of unforeseen consequences of designing high technology systems, as solving one technical hurdle leads to the revelation of ten-fold more, the revealing often taking dramatic form such as an aircraft crash or network failure. These system collapses usually seem to come with little warning or means of aversion.

Great questions underlie what drives or enables complexity and what factors contribute to the decline or collapse of a complex system. Despite apparent differences between systems, there are common principles of complexity that can be applied to most or all complex systems. It is the aim of this research to provide a better understanding of how to approach complexity, establish firm linkages between measurement of complexity and measurement of functionality, determine system design strategies for resilience in complex systems, and develop methodologies to guide systems engineers.

1.1 Motivations

Modern tools of civilization, particularly military systems, have become significantly more complex over time. In prior centuries, the essentials of warfare were assumed stable and well understood: keep the men and horses healthy, maintain national support, and apply enduring principles of warfare. The results could be predicted within a limited range of scenarios.

In today's world, a vigorous solar storm or well-placed Electromagnetic Pulse (EMP) can take down the electric grid for months or years, compromising military acquisition, logistics, or national integrity itself [50]. Anti-Satellite (ASAT) weapons could cripple the U.S. military's navigation, targeting, and communication systems. Geo-political realignment could reduce U.S. oil access by 80%, almost completely eliminating the country's ability to wage conventional war to a much greater degree than faced even in World War II [32]. In short, critical system dependencies exist where none did before since modern life and modern warfare have created novel systems of increased complexity (e.g. the national electrical grid, the internet, the Global Positioning System, mobile device technology, oil-based infrastructure, mechanized agriculture, Just-In-Time logistics systems, etc.). These system dependencies exist on top of the dependencies common with the past such as procuring food supplies. The immense benefits of these systems are clear; less clear are the non-linear vulnerabilities that emerge as these complex systems migrate from novelty to normalcy.

The modern world's adoption of these complex systems has outpaced our capacity to analyze and prepare for critical vulnerabilities that can lead to cascading collapse, affecting military systems and beyond. The key motivation for this research is to develop theory and methodology to assist in designing resilient architectures, thus providing the best opportunities to avoid adopting systems and strategies that introduce new critical vulnerabilities.

1.2 Problem Statement

Designing for resiliency is most effective early in the systems engineering process, a point where, unfortunately, the least amount of system detail is available or even considered. As concepts progress from low fidelity to high fidelity, certain design features become "locked in" and increasingly difficult to change. It is not enough to develop theory and methodology to assist in complex systems design, it must be employable easily and early in system design. Therefore a second key motivation is to adapt the theory and methodology to a quick turn effectiveness analysis capability. The methods developed in this research must be scalable to all levels of fidelity, from initial concept description to analysis of fielded systems.

1.3 Research Objectives

The primary goals of this research are to develop a method to measure the functionality and complexity of a system in a practical manner and use that methodology to assess the resilience of the system. First, it is important to articulate what is meant by functionality and complexity, as there is significant variance across the literature. The study of the literature will form a basis for deriving applicable measures and methodologies. The next objective is to assess the impact of constrained energy and information in theory. The final objective is to demonstrate the theory with using simulation to begin validatating the theoretical conclusions.

1.4 Research Questions

It is hypothesized that higher functionality and higher complexity drive increased system resilience subject to constraints on availability of energy and information. In proving/disproving this hypothesis, the follow questions need to be answered:

- How can functionality be measured?
- How can complexity be measured and what is its relationship to functionality?
- What is the effect of reduced energy availability on functionality and complexity?
- What is the effect of reduced information on functionality and complexity?
- How can knowledge of functionality and complexity be applied to the systems engineering process?

1.5 Method Overview

The overall approach in meeting the research objectives and answering the questions in the problem statement is described below. Chapter II organizes and analyzes the existing literature on functionality and complexity, forming the basis for definitions and metrics related to functionality in Chapter III. The functionality metrics are developed further and used to explore the impact of changes in energy availability on functionality. (Note: Chapter III is a wholly contained paper submitted to the International Council on Systems Engineering (INCOSE) *Journal of Systems Engineering* [18].)

Chapter IV extends the discussions of functionality and energy to complexity and information, leveraging insights from information theory and the Law of Requisite Variety (LRV). This enables a complete theoretical treatment of functionality, complexity and assessing resilience under changes in energy and information. (Note: Chapter IV is also a complete journal paper which will be submitted to the Institute of Electrical and Electronics Engineers (IEEE) *IEEE Systems Journal.*)

A simulation is described in Chapter V. Simulation results are also found in Chapter V, which are used to validate the theory. Finally, the theory and analysis

are used to draw conclusions and make recommendations for future work in Chapter VI.

1.6 Research Contributions

- This research demonstrated a practical methodology for assessing the functionality of dynamic sytems (as a proxy for complexity), which can be used to compare systems in common scenario sets, measure historical functionality, or set bounds on the maximum functionality.

- This research established a theoretical basis for guiding the design of systems that are resilient to conditions of limited energy and information.

- This research coupled several areas of study into one coherent theory, drawing from complexity theory, information theory, systems theory, systems engineering, reliability engineering, and systems architecture.

II. Literature Review

The focus of this research is to develop a practical methodology for assessing the functionality and complexity of dynamic systems. The development of this methodology and supporting theory leverages complexity theory, information theory, systems theory, systems engineering, reliability engineering, and systems architecture. This research drew most heavily from the diverse fields of complexity theory and systems engineering. As this research will demonstrate, the route to practical measurement of complexity is through functionality.

2.1 Systems and Functionality

2.1.1 Defining Systems.

Before developing the methodology to assess aspects of complex systems, it is necessary to precisely define what is meant by a "system". In the systems engineering community, several popular definitions for the term have emerged in recent years.

The Department of Defense (DoD) defines a system as "A functionally, physically, and/or behaviorally related group of regularly interacting or interdependent elements; that group of elements forming a unified whole". [51] Further, it defines System of Systems (SoS) as "An SoS is defined as a set or arrangement of systems that results when independent and useful systems are integrated into a larger system that delivers unique capabilities". [51]

System architectets Maier and Rechtin define system as "a set of different elements so connected or related as to perform a unique function not performable by the elements alone". [46] This definition does not explicitly account for scale or environment. The International Council of Systems Engineering (INCOSE) adopted a similar definition adopted from Maier and Rechtin:

> "A system is a construct or collection of different elements that together produce results not obtainable by the elements alone. The elements, or parts, can include people, hardware, software, facilities, policies, and documents; that is, all things required to produce systems-level results. The results include system level qualities, properties, characteristics, functions, behavior and performance. The value added by the system as a whole, beyond that contributed independently by the parts, is primarily created by the relationship among the parts; that is, how they are interconnected." [36]

2.1.1.1 Kuras Definition of System.

An expansive treatment by Kuras worked to establish an improved definition of a system that captures the multi-scale nature of systems, structural and dynamic aspects of systems, and accounts for subjective and objective qualities in the human perception of systems. [43] His basic definition of a system is "a collection of properties and relationships that is both a whole and a part of a more expansive whole." [43] This definition emphasizes the notion of how a system exists at multiple scales and thus fits with the motivations of the research topic.

Kuras takes his definition much farther and more thoroughly than the straightforward version above. Before presenting the expanded definition it is necessary to define a number of terms.

A conceptualization is a subjective interpretation of reality by an observer. A conceptualization has: a Field of View (FOV), which is a measure of the expanse or inclusiveness; a resolution (RES), the degree to which portions of the conceptualization can be distinguished; and one or more patterns, the content of a conceptualization.

The concept of field-of-view is well-understood in the realm of systems architecture. Changing the observer's perspective of the system yields new insights. Linked together, multiple views portray a more complete understanding of the system. [46]

Patterns are a rich topic of discussion, but are in essence a structure (conceptualization) of a system as perceived by an observer. A key point here is that the focus

(and system scale) can enable the perception of a different pattern of a system. [43] For illustration, consider the dramatic change in perspective when focusing on one pattern or the other in depth-perception-based holograms.

A holon (H) is a entity that is both a whole and part of a more expansive whole (see the simplified Kuras definition for system above), which is used to distinguish those patterns that refer to a particular system.

Using equation 1, a system $\mathfrak{s}$, is a collection of parts that is: represented (approximated) by conceptualizations at one or more scales μ through ν; is always conceptualized as a holon at each scale at which the system is conceptualized, such as H^{μ} ; is always conceptualized at each scale as objects (E_i) and relationships (REL_j). P_m denotes a property and REL_n denotes a relationship, where $m \in M$ and $n \in N$ are index sets enumerating all properties and relationships in all possible conceptualizations. H denotes the holon that explicitly distinguishes the parts of the system from the parts of its environment and that explictly provides the cohesion of its parts as a whole. The notation in equation 1, per Kuras, indicates the fact that $\mathfrak{s}$ may be conceptualized at multiple scales. [43]

$$\begin{aligned} \mathfrak{s} &\equiv \{P_m; REL_n\}_{M,N}; H \\ \mathfrak{s}^{\mu} \in \{S^{\mu}\}, S^{\mu} &= \{E_i; REL_j\}_{I^{\mu},J^{\mu}}; H^{\mu} \\ &\vdots \\ \mathfrak{s}^{\nu} \in \{S^{\nu}\}, S^{\nu} &= \{E_i; REL_j\}_{I^{\nu},J^{\nu}}; H^{\nu} \end{aligned} \tag{1}$$

For each scale-specific approximation in equation 1, the relevant modalities of focus, field of view, resolution, and frames of reference are explicitly noted. S^{μ} denotes all time invariant (with respect to frame of reference) parts of the scale-specific approximation. Simple objects and relationships can be aggregated as compound objects consistent with reductionism and determinism.

2.1.2 Defining Functionality.

For the purposes of engineering, the definition of "system" typically includes reference to performance or function. The definition used by Maier and Rechtin for system is "a set of different elements so connected or related as to perform a unique function not performable by the elements alone." [46] This definition does not explicitly account for scale or environment. INCOSE adopted a similar definition. [36]

A representative definition for function is "a task, action, or activity that must be accomplished to achieve a desired outcome or provide a desired capability." [42] While accurate, more precision is required before approaching a measureable quantity.

Stryker [63] defines function as "[a] technical process involving energy, material, and/or signals being converted and or channeled." The energy, material, or signals acted upon are defined as flows.

It should be noted that there is an established body of work known as Function Point Analysis (FPA) which is defined as measuring the business functionality that an information system provides to a user [38]. However, this field applies exclusively to software, as opposed to dynamic systems, and it applies only to estimating the cost, duration, and amount of resources required to develop a software project. This research seeks to establish and analyze functionality with a systems engineering view, which is a very different context than the business motivations of FPA.

Stryker defines functionality as "the number of functions to be performed by a system." [63] INCOSE defines functionality as "a set of attributes that bear on the existence of a set of functions and their specified properties". [35] Buede [11] defines functionality as "a set of functions that is required to produce a particular output". In addition, Simple Functionality is defined as "an *ordered* [sic] sequence of functional proceses that operates on a single input to produce a single output." Also, Complete Functionality is defined as "a complete set of coordinated processes that operate on

all of the necessary inputs for producing a specific output".

2.2 Fundamentals of Complexity and Complex Adaptive Systems

A complex system is said to exist on the "edge of chaos", where the system is neither chaotic nor static. It is not wholly predictable, but it still exhibits patterns allowing insight. A key distinguishing feature of complex systems versus merely "complicated" systems is the level of reducibility. [48] Complicated systems are composed of relatively independent parts, but complex systems' components are dependent on one another. Maier and Rechtin [46] define complex as "composed of interconnected or interwoven parts". Removing a component fundamentally alters the structure of the system. It is the difference between moving furniture and moving a wall.

2.2.1 Chaos.

A classic example of chaos theory is that a butterfly flapping its wings in Peking can change the weather in New York a week later. Restated, the predictions of the weather system can rapidly be rendered useless with but a slight change in initial conditions.

Climate, however, is the deep structure of the weather system. Precise predictions are limited by chaotic behavior, but it takes significant stimulus to upset the climate. Patterns (structure) are formed from feedback within the climate/weather system and with external systems, such as humans.

In a chaotic system, initially close points will diverge exponentially as the system operates, thus making the system unpredictable. This chaos can derive from more than one source: either the system is unpredictable because it is stochastic or the underlying structure of the system dynamics is non-linear.

In deterministic chaos, the rules of the system are clear and unambiguous. An

initial point is mapped to another point that requires increased precision. After many cycles, the result is unpredictable because the necessary precision exceeds our abilities. As an example, consider the gravitational 3-body problem: it is based on completely deterministic rules, but after a finite number of iterations, the calculation becomes unmanageable. The system diverges exponentially as the required precision exceeds that of whatever computer is used, and small errors come to dominate the dynamics rather than any initial regular motion. [58]

The purpose for exploring deterministic chaos is to illustrate that we need not invoke stochastic processes to produce chaotic (read: unpredictable) behavior. Rather, it shows that chaos is a deep property of non-linear systems, which are further complicated by the underlying uncertainty of the universe. In a deterministic chaotic system, a purely random choice of initial point has zero probability of infinite survival. In a stochastic chaotic system, *any* initial point has zero probability of infinite survival.

2.2.2 Flickering indicators, tipping points, and rapid state transitions.

Collapse can be defined as a rapid reduction in complexity, either into chaos or stasis. The challenge for those studying complex systems is to predict when a collapse might occur, which might further lead to the ability to control (cause or prevent) collapse.

Flickering indicators, also known as squealers, are possible predictors of collapse. There is clear evidence that systems characterized by a bifurcation produce squealers shortly before rapid state transitions. [57] In a fold bifurcated system, the equilibrium curve is not linear but "folds back" on itself to produce two separate continuous functions across an unstable gap. As a system in equilibrium adjusts its equilibrium state in response to external conditions, a fold bifurcated system will approach a

critical tipping point – upon reaching it the system state will rapidly transition across the gap to the other continuous curve.

In such systems, squealers appear as the system slows down in its rate of response to external stimulus. This change in recovery rate (one squealer) starts far from the bifurcation point and decreases smoothly as the transition is approached. This slowing is the result of the eigenvalue for the rate of change of the system state approaching zero at the bifurcation.

This slowing down phenomenon of fold bifurcated systems yields other interesting squealers. As a system slows down (approaches the threshold), its autocorrelation coefficient will increase because each state becomes increasingly like its past state. The variance also increases because perturbations do not decay. Other potential squealers are related to spatial patterns. There is also evidence that systems with bifurcations other than the fold type exhibit similar behavior. [57]

2.2.3 Collapse.

The tipping points and rapid state transitions of the previous section correlate with what is commonly known as collapse. Intuitively, most people understand what is meant by the collapse of a system. In discussing the collapse of a civilization, such as the Western Roman Empire, it is clear that a highly complex society transitioned to a fractious, less complex one. This understanding motivates anthropologist Joseph Tainter's definition of societal collapse as "a rapid reduction in complexity" [65].

United States Air Force (USAF) Lt Col Daniel Marticello used Tainter's work to qualitatively demonstrate the trend towards collapse of the US Air Force acquisition system [47]. As the research explored in this paper in part links Tainter's work on complexity to physics-based notions of complexity, it could complement Marticello's work with a quantitative approach.

As stated, the primary goals of this research are to develop a method to measure the complexity of a system in a practical manner and use that measure to assess the stability or resilience of the system. Of primary interest for application is assessing the resiliency of system architectures in energy-constrained and information-constrained environments. Such a method could have a wide variety of applications, however, including anticipating collapse of systems (such as the aforementioned USAF acquisition system).

2.2.4 Complex Adaptive Systems.

Sheard and Mostashari [60], discuss some aspects that describe a complex system. These include: being composed of many autonomous components (agents); being self-organizing; displaying emergent macro-level behavior; and adapting to the environment (fitness landscape). Each of these aspects will be discussed in further detail in later sections.

Complex Adaptive Systems (CAS) can be characterized by the following properties: the system consists of a network of many agents acting in parallel; it has many levels of organization; it anticipates the future; and it has many niches which can be exploited by agents adapted for the purpose. [67] All of these properties, which will be investigated further, serve to produce emergence – a collective property of a system that cannot be predicted by examining each agent individually.

Important to CAS is the concept of dynamic equilibrium, or homeostasis. CAS are living systems (usually not just figuratively), with constantly moving processes. [48] If we consider cell osmosis, there is a point referred to as equilibrium where water concentration is stable on both sides of the membrane. However, this is dynamic equilibrium – if it is static equilibrium, there is no fluid exchange and the cell is therefore dead. This can be extended to all CAS: when we speak of stability, we're

speaking of homeostasis – the system is always in motion.

Why is this important? With a dynamic system, something must always occur (or be occurring) for the system to operate. If that something is interrupted or modified it can have profound effects on the system's seemingly stable behavior.

2.2.5 Fractals.

Fractals derive their name from their non-Euclidean nature: instead of being defined by integer dimensions (1-D for a line, 2-D for a square, etc.), fractals must be described by fractional dimensions. Examples of fractal geometry can be seen in clouds, coastlines, trees, and many other natural phenomena. Fractals also exhibit self-similarity (see Figure 1), which enables useful mathematical modeling of these shapes. Fractals tie strongly with the concepts of resolution and field-of-view discussed earlier regarding the definition of "system" by Kuras [43].

Figure 1 shows a classic fractal known as the Koch curve. For each iteration of construction, the middle third of each line segment is replaced with two lines of length 1/3 of the original (x=3), which creates 4 line segments from the original one (q=4). With each successive iteration, the length of the curve approaches infinity. [58]

The length of a curve $L(r)$ is given by the product of the number of line segments required to cover it times the size of those line segments. For a Euclidean curve, as the line segments shrink, the product $N \cdot r = L(r)$ approaches a finite limit. However, for fractals this product diverges, since we encounter ever smaller detail. Instead there is a critical exponent D_H, known as the Hausdorff dimension, for which $N \cdot r^{D_H} = L(r)$ remains finite. In the limit, $D_H = \lim_{r \to 0} \frac{logN}{log\frac{1}{r}}$.

Looking at a generic fractal generator, the number of pieces N is proportional to q^n, and the n^{th} line segment has length $r = \frac{r_0}{x^n}$. The equation for the Hausdorff

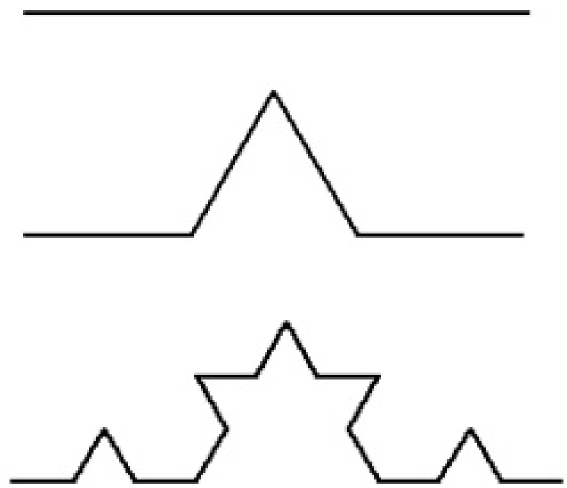

Figure 1. Initial iterations for constructing a Koch fractal

dimension becomes $D_H = \lim_{n\to\infty} \frac{log\ q^n}{log\ x^n - log\ r_0} \to \frac{log\ q}{log\ x}$. A Euclidean curve has $D_H = 1$, a Euclidean area has $D_H = 2$, and so on. [58]

Fractals are important because they describe the deep structure of many complex systems. Coastlines (and national borders) are fractal shapes. Functions in the brain, such as sound processing, exhibit a fractal nature. Brownian motion (a stochastic fractal) describes not only the motion of particles on the surface of water, but the structure of mountains. Fitting fractals to CAS, such as warfare systems, could yield useful insight and will be explored in a later section.

2.2.6 Modeling Complex Adaptive Systems.

2.2.6.1 Agent-based techniques.

Although often tied to Complex Adaptive Systems, agent-based models are just one, albeit powerful, method available for exploring CAS. As discussed above, there are many useful mathematical tools appropriate for CAS modeling.

Agent-based modeling uses collections of simple (relative to the system under ex-

amination) autonomous “agents”. Agents can be animals, soldiers, molecules, atoms, cells, stock brokers, or organizations. We can abstract the definition further to apply to functions, ideas, or innovations. It should be recognized that any agent, particularly humans, is made up of many sub-agents: cells, molecules, atoms, sub-atomic particles, all of which affect each other to produce complex emergent behavior at various functional levels.

The key to agent-based modeling is establishing an appropriate rule framework. The ‘edge’ in the edge of chaos is not in phase space but in the space of rules. [48] An agent is faced with constant decisions on courses of action balanced between all the rules in its set. One rule-mixing strategy may produce elegant emergent behavior while another causes uncontrolled divergence, and the results will vary as the environment changes.

2.2.6.2 Networks.

Network modeling is another approach to managing CAS. Formal treatment of networks leads to realistic models that are amenable to mathematical analysis. For example, networks such as the World-Wide Web, metabolic pathways in cells, telephone calls, and sexual contacts approach power-law distributions. [48] Networks in general are robust against random failures, but targeted failures can have powerful effects – provided the targeter has a good understanding of the network structure.

Some argue (convincingly) that all agent representations discussed in CAS are networks – in rule-based agent systems, agents are the nodes and the rules are the connections; in genetic algorithms, genes are the nodes and they are linked by the processes of recombination; and even autocatalytic molecule systems have polymer compounds as agents and catalytic molecules as links. [67]

There are several types of networks, some of which lend themselves more easily to

analysis. Some of these main types include Small-world, Loop, and Pack networks.

In a Small-world network, each node is initially connected to a set of neighbors. Those connections are then randomly severed and new connections to random nodes are established. This leads to a "6 degrees of separation" model where there are trivially few connecting nodes between any two given ones. [48]

In a Loop network, agents live on a circle and are connected to their nearest neighbors in either direction. Agents look to their nearest neighbors in deciding actions. In such a network, behaviors propagate from neighbor to neighbor, which simplifies analysis. When this simple structure is extended into two dimensions (imagine a continuous checkerboard – the top and bottom ends are brought together, then the left and right), the resultant torus topography is known as a Grid network.

In a Pack network, each node on a loop network is replaced with a collection of agents. Each of these agents is connected, and each agent is linked with one other agent, usually in the corresponding location in the next pack.

An N-Loop network consists of N loop networks where each agent has additional connections to agents on other loops. An example of this representation is an agents family, friends, online contacts, and schoolmates networks.

These different network structures produce very different results for agent behavior. When applied to segregation models (mixing of different agent types), Pack networks produce the highest levels of segregation and Grid networks the least. Defining conformity as the percentage of agents choosing one of two possible actions, Pack networks also produce the highest conformity and Loop networks the lowest. The power of using these structures is that these claims can be proved mathematically, inviting more rigor into CAS analysis and development.

2.2.6.3 Communication and cooperation.

Inputs that agents receive often come not just from the surrounding environment, but also other agents. This invites opportunity for agents to manipulate some aspects of other agents' behavior. [48]

The Prisoners Dilemma is a classic illustration of game theory and strategic cooperation. In the scenario, two prisoners are faced with the decision to cooperate with their isolated co-conspirator or defect and sell out their partner to the authorities. Payoff is determined according to the matrix $A = \left(\begin{smallmatrix} R & S \\ T & P \end{smallmatrix}\right)$ where $T > R > P > S$. Thus, if both prisoners cooperate, they receive a medium reward value. If both defect, they receive a small reward. If only one defects, the defector is richly rewarded and the cooperator receives little or no reward. A single play of the game yields the solution that the optimum course of action is to defect. Thus, through strict logic (via Nash Bargaining) [49], both agents have selected the least desirable scenario. This is also true by extension to N agents, with total rewards calculated as $R_C = \dfrac{R \cdot N_C + S \cdot (N - N_C)}{N}$, $R_D = \dfrac{T \cdot N_C + P \cdot (N - N_C)}{N}$, where N_C is the number of cooperating agents, R_C is the reward for cooperators and R_D is the reward for defectors. The value for R_D is always greater than for R_C. [28]

Developing a strategy for multiple rounds, however, is much more complex. Given an appropriate rule set, agents now have a history, a memory, and a running tally of reward. In the first set of tests by Miller and Page [48], agents were pitted against one another in a series of games, then allowed to adapt their strategies (via genetic algorithm [31]). Some correlation between high variability in potential payoff and high strategic complexity was found (producing more dynamic adaptation), but defection was still the dominant strategy.

The results are intriguing, but are missing a key component of (non-isolated) intelligent agents in the real world: communication. In further tests of game-playing

agents, Miller and Page [48] introduced a framework with which agents could communicate. Starting with a set of known communication tokens, none of which are preassigned any special meaning, agents exchange and receive a set of tokens prior to each play of the game.

This framework gave rise to very dynamic emergent behavior observed by Miller and Page. The ability to communicate gave rise to strong bursts of cooperation, followed by the emergence of mimics, and long periods of mutual defection. [48] Because the agents adapt and improvise (via the genetic algorithm rules), the system exhibits periodic stochastic escape from the mutual defection attractor. Such behavior shows the importance of information flow on system dynamics.

2.2.6.4 Organizations and hierarchy.

Organizations exist to solve problems more effectively than an individual agent. It can be proven that a deterministic organization can be developed to perfectly solve one deterministic problem. [48] However, given the full set of problems, any organizational structure will be equally accurate in producing a solution. It is therefore demonstrated that organizations that dont tailor their structure to the problem at hand are no more competitive than any other. In addition, an organization attempting to span multiple problems must make decisions on acceptable levels of solution quality (fitness), since it is not possible for one organization to perfectly solve more than one.

Of course, no organization exists in isolation. Every hierarchy fits within other higher-level, overlapping, or reshuffled hierarchies. In addition, organizations are in constant competition both with external hierarchies and internal forces. For an organization to survive in an evolutionary environment it must constantly adapt its structure to solve multiple problems and compete with opponents, all while ensuring its structure aligns with the overall hierarchy. Exploring organizational rules appears

a rich area for research, and would be of particular interest to military commanders.

2.2.6.5 Warfare agents.

Using agent-based modeling can give us clues to what types of tools we should take into battle and in what ways to employ them across a range of adversaries with differing capabilities or resources.

Military doctrine increasingly recognizes the opponent as a living and adapting warfighting system. Old sciences used for developing strategies are unable to keep up with these advanced principles of doctrine because they fail to model the truly important part of warfare – the complex interactions. Principles such as maneuver warfare, Warden's Five Rings systems approach, and strategic effects require something more.

That something more is found in agent-based models. Individual soldiers can be modeled with simple rules and the resultant large scale dynamics can be simulated repeatedly. Differences in training, skills, resources, and strategy can all be encoded and tested.

Advanced warfare models such as the Hierarchical Interactive Theater Model (HITM) and the Irreducible Semi-Autonomous Adaptive Combat (ISAAC) model have demonstrated effectiveness in accurately modeling warfare scenarios, and have proved extremely adaptive to changes in the simulation environment. [12] HITM, for instance, illustrates the importance of rapid maneuver particularly when faced with resource constraints.

2.2.6.6 Fitness landscapes.

Every agent operates within an environment. This environment is made up of topological features in addition to other agents operating at multiple functional levels. It is within this environment that an agent must seek to thrive to achieve maximum

fitness. This environment can therefore be described as a fitness landscape.

The environment of a complex system is often of greater complexity than the system itself. The environment also constantly changes, due to influences internal and external to the system. [60] As each agent develops, it changes the fitness landscape of all other agents in the local network.

A fitness limit can be defined as a driving stress on a population that upsets the balance between uniformity and variety in either direction, thus leading to extinction. [23] In warfare modeling, a fitness limit is characterized by an either paralyzed or fragmented force; the opponent (in combination with the fitness landscape, or environment) drives the force to failure (extinction). There are multiple philosophies from military theorists on how a force might accomplish this, but one of the most relevant and powerful for agent-based techniques is found in the simplicity of Boyd's OODA Loop (discussed in the next section).

In engineered systems, it is usually an open question as to the appropriate determinant of the fitness values as it is very easy to introduce designer bias. In warfare systems, it seems most appropriate that fitness measurements should be based on the commander's intent.

2.2.7 OODA Loops.

One increasingly popular theory of warfare is based upon the little-published work of USAF Col John Boyd [19]. At the heart of the theory is the idea of continual loops which follow the process of Observe-Orient-Decide-Act (OODA), with multiple feedback points and internal complexities [10].

Boyds visual depiction of the OODA is shown in Figure 2 [10]. Notice that it is not one single loop, but multiple processes of internal adjustment and correlation.

Many models assume that agents perform all elements of the OODA cycle simulta-

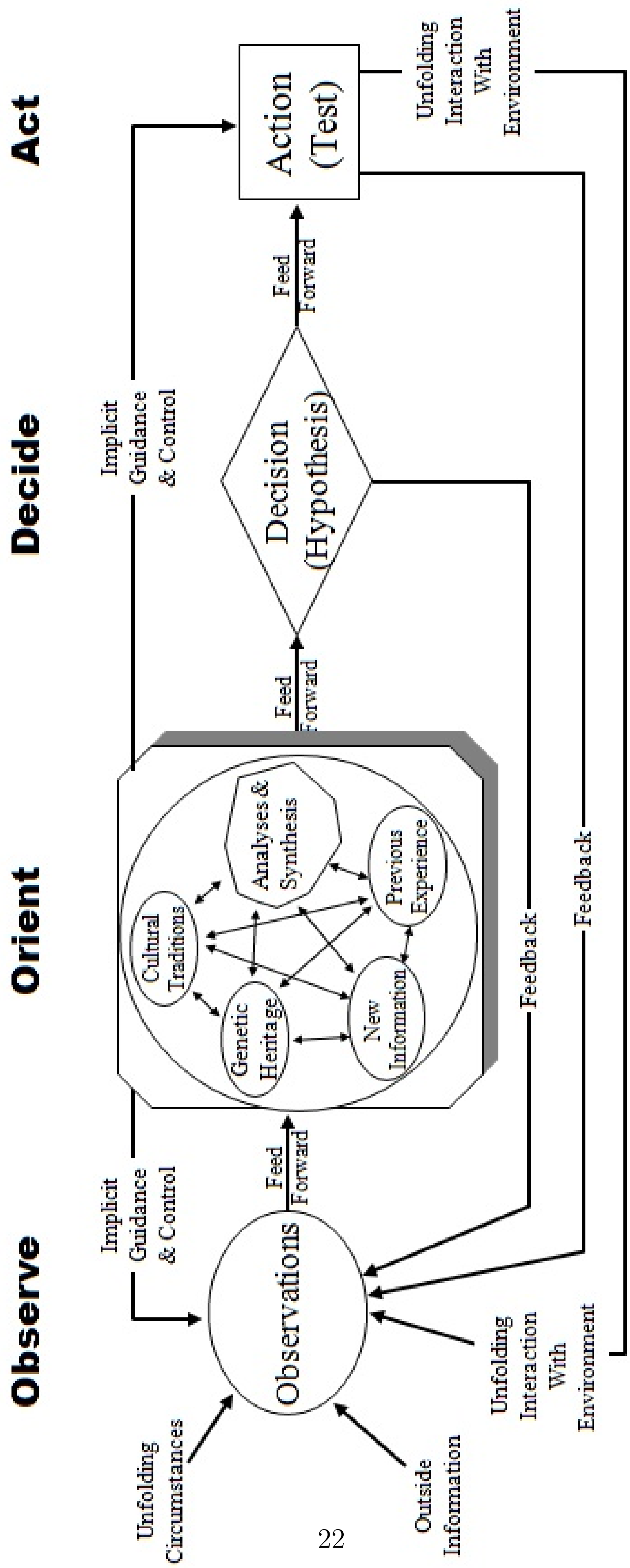

Figure 2. Boyd's OODA Loop sketch.

neously. This approach removes key timing behavior from the model which in the real world is crucial to any complex activity. In practice, there is coordination of OODA loops, which should be the principle focus for the designer or military commander. The HITM (discussed earlier) encodes OODA loops within each soldier agent, and the effects of varying OODA loop coordination can be seen. [12]

2.2.7.1 Fractals in time.

OODA loops are present at all levels of an organization. This begs the question, what organizational structure will yield the fastest OODA cycle? Gustavsson and Planstedt discuss the concept of fractal OODAs. [29] Recall that for geometrical shapes, $N \cdot r^{D_H} = L(r)$ [58]. The equivalent representation for time is $N \cdot t^{D_H} = L(t)$, where t is a segment of time. For an ordered fractal, each subprocess must be x times faster than the next-higher level process, or for the n^{th} cycle $t = \frac{t_0}{x^n}$. Similarly, the number of cycles N is proportional to q^n. This leads to the identical result for physical space, $D_H = \frac{log\ q}{log\ x}$.

One interesting (if obvious) result is that by increasing the number of subprocesses one can compensate for a slower OODA loop time while holding D_H constant. This makes sense as having more agents allows you to gather more information and execute actions faster. But there must be an upper limit where internal complexity negatively impacts the system, and this is where Complex Adaptive Systems approaches come into play.

2.2.8 Complex Adaptive Systems Engineering.

Systems Engineering of complex systems must be considered more as designing the engineering and management mechanisms than a holistic detailed design of a system (which is precisely the approach of Systems Architecture). The principles of CAS

force a focus on the interactions between systems and how those interactions drive adaptation, truly evolutionary design. Since we cannot predict with certainty how a complex system will evolve, we cannot afford to attempt to design with certainty. Such an effort leads to false impressions of precision and non-adaptable systems.

A counter-intuitive concept from Systems Architecture is the hourglass model: forcing system standardization at a targeted inflection point enables increased adaptability and divergence of innovation. The clearest example of this is the Internet Protocol (IP), which allows a diversity of hardware and link technologies on one side and diverse applications on the other. [23] The implications from this are intriguing: levying a hard constraint on a very specific point of the system drives increased emergence. This is a key clue to unraveling the mystery of how to influence adaptive systems, that one must grasp the deep structure of the problem to identify the critical points. (The parallel here with the language of maneuver warfare is not trivial; driving emergent warfare behavior hinges on understanding both opposing and friendly force structure, and using the knowledge to apply pressure at the point of greatest effect.) No model can fully describe the system under consideration; there is always a mismatch between models and reality. [9] It is the constant job of the researcher, analyst, or strategist to reduce this mismatch by creating new models as the old ones lose their usefulness.

2.3 Thermodynamic Entropy

In order to fully explore approaches to measuring complexity, it is necessary to review the concept of thermodynamic entropy. Boltzmann defined entropy for a perfect gas changing states isothermally according to equation 2, where N is the thermodynamic entropy, k is the Boltzman constant, T is the temperature, E_ψ is the Gibbs energy, h is the total heat of the system, and X is the set of possible system

states. [56]

$$H = -k \int_X \frac{(E_\psi - h)}{kT} exp(\frac{(E_\psi - h)}{kT}) dx \qquad (2)$$

As the quantities of molecules involved in such a system become large, this deterministic expression is replaced with a probabilistic one. The probability of the system being in state x is defined according to equation 3. This definition yields the simplified expression for entropy in Equation 4. [56]

$$p(x) \equiv exp(\frac{(E_\psi - h)}{kT}) \qquad (3)$$

$$H = -k \int_X p(x) ln(p(x)) dx \qquad (4)$$

Entropy may be further generalized to apply to any physical system. Thus, it can be seen that entropy is related to the state space of a system and the probability distribution of that state space. Entropy is a statistical measure based on a mechanical process. The second law, which states that entropy for closed systems must increase or remain constant over time, is a statistical law. That is, there exists a finite probability that a system will spontaneously reduce its entropy, but over a long time horizon ($t \to \infty$) the probability of such an event approaches zero.

Kittel [41] lists several basic processes that increase entropy in a closed system, which is analogous to increasing the size of the system's state space.

2.4 Measuring Complexity

A difficulty in complexity theory is the lack of a clear definition for complexity, particularly one that is measurable. In many cases throughout the literature, it seems the authors are speaking of different or even mutually exclusive phenomena. The most

apt summary of the issue is the tongue-in-cheek assertion by complexity researcher Seth Lloyd: "I can't define it for you, but I know it when I see it." [20]

A root cause in the lack of unified complexity definitions is that there are in fact several types of complexity. The first formal treatment of complexity focused on algorithmic complexity, which reflects the computation requirements for a mathematical process. [20] Senge and Sterman include also dynamic complexity, which is primarily characterized by difficult-to-discern cause-effect relations. [59] [62]

One of the most workable definitions is that of thermodynamic depth, which actually seems to unify the two complexity camps. Thermodynamic depth as complexity asserts that complexity is a "measure of how hard it is to put something together". [45] [22] There are several variations on this approach, with the commonality that complexity disappears for both ordered and purely random systems. [44] [20]

Bar-Yam defines the complexity of a physical system as the length of the shortest string, s, that can represent its properties. This can be the result of measurements and observations over time. [5]

An energy-based metric is proposed by Chaisson. By measuring the energy rate density in equation 5, where Φ_m is the energy rate density, E is energy flow through a system, τ is the time epoch, and m is system mass, Chaisson obtains results that correlate well with other notions of complexity. [13]

$$\Phi_m = \frac{E}{\tau m} \tag{5}$$

However, the energy rate density metric has some drawbacks. By normalizing with respect to mass, this metric produces incorrect results for complexity when comparing some systems. For example, suppose an electronic brain is built to mimic the operation of a human brain. The human brain may process energy at the same rate as a theoretical electronic brain, but due to differences in basic materials (i.e.

the weight of neurons vs. semi-conductors), the two systems, which most theorists would recognize as identically complex, could have vastly different Φ_m values. Thus, by normalizing with respect to mass instead of function, the Φ_m metric produces incorrect results for the relative complexity of systems.

A practical difficulty in using the Φ_m metric is determining the appropriate mass and energy to use. In measuring the Φ_m of a civilization, Chaisson uses the mass of humanity and the total energy processed by the civilization. However, the total energy of a civilization does not flow through only its humans, but also its machinery, beasts of burden, vehicles, etc., the mass of which is a difficult quantity to measure.

2.5 Energy Scarcity

As discussed in Chapter I, there is significant risk to modern combat systems under the scenario that friendly energy resources become severely restricted. This could be due to budget economics, geological constraints on fossil fuel production, or foreign suppliers collaborating to withhold energy sources.

While the energy landscape for national, international, and military interests is complex and varied, for operations that are directly relevant to warfare there is one clearly dominant energy source – oil. The problem could be greatly simplified by focusing on this one energy source without losing great accuracy of the results. These direct (first-order) activities include logistic transport within the nation's borders, transportation to the Area of Responsibility (AOR), transportation within the AOR, power for field equipment, and power for Forward Operating Bases (FOB). (Secondary levels of warfare support could have implications for national level policy and invite additional analysis; these include energy supplies for manufacturing, force readiness, deployment/business travel, and so forth.) [32]

Oil yields power through its energy density. The brute force approach is to use this

fact to overpower systems, where there is little need for energy adaptability provided there is a relatively unconstrained oil supply. But being forced to use less oil or an energy system with lower energy density eliminates brute force as an option. Instead, it drives the system to seek ways to adapt and respond rapidly to an opponent, to flow around their brute force systems and strike vulnerable locations.

Providing a fitness landscape consisting of energy sources, blue and red forces, geologic features, and other relevant data could enable an agent-based system to explore new niches for an energy-flexible force. A competitive and adaptable warfare system makes use of previously inaccessible and hidden areas of the fitness landscape. A UAS with the ability to sense thermals and soar has captured an energy niche inaccessible to an F-22. A methodology to rapidly identify resilient system architectures from a wide pool of candidates against a diverse environment can assess the utility of using such niches.

2.6 Information

The DoD has put significant emphasis on Intelligence, Surveillance, and Reconnaissance (ISR) systems dedicated to the collection and processing of information. Complexity Theory recognizes strong relationships between information and complexity, but open questions remain: Does information drive or enable complexity? What role do changes in information processes have in system collapse? How can information overload be managed?

Information overload in particular is a key concern for the military [40] [53], resulting in billions of lost dollars and lost capability.

Whereas establishing a link between complexity and physical processes required reference to thermodynamic entropy, understanding complexity's link with information requires a detailed treatment of information entropy and principles of information

theory. We must also consider what type of information is of concern.

Per Gray [27], in information theory, a dynamical system $(\Omega,\ B,\ P,\ Z)$ is measured using a finite alphabet mapping, $f : \Omega \to A$, where Ω is a measurable event space, B is a particular collection of subsets of Ω, P is a probability measure, Z is a transformation of the event space such that $Z : \Omega \to \Omega$ (such as time), and A is an alphabet. It should be noted that an alphabet is not limited to something akin to our familiar A through Z; a set as vast as the complete set of words in the English language can constitute an alphabet in this treatment.

The entropy of a random variable, a, from that alphabet may be defined according to equation 6, where p_f is the Probability Mass Function (PMF) of f. [27] This expression is clearly analogous to thermodynamic entropy (equation 4) . The difference is the thermodynamic states, Ω, have been mapped to a set of information states, A.

$$H(f) = -\sum_{a \in A} p_f(a) \ln(p_f(a)) \tag{6}$$

Just as thermodynamic entropy is regarded as a measure of the uncertainty of the state of a system, information entropy is considered a measure of the uncertainty of the information about the state of a system. Using this understanding of the nature of information uncertainty, it is possible to understand the links between complexity and information. Bar-Yam defines complexity of a system as the length of the shortest information string s that can completely represent the properties of that system. This complexity value is equivalent to the information content of a system. [5]

2.6.1 The Law of Requisite Variety.

Ashby's Law of Requisite Variety (LRV) defines variety as the number of elements (or states) in a set that can be distinguished. [3] As an example, the set {b r b a b r} has a variety of 3 letters. Variety can refer to specific events, scenarios, or actions. In

order for a regulator R to control a set of disturbances D to yield a specific favorable outcome from set E (the set of all desired outcomes) it must have at least as many possible responses as there are disturbances. [3] If the same response was applied to two different disturbances, there would be two different values in the outcome set, E (a specific outcome is determined by both the disturbance and response). This is uncontrolled regulation, since there is no control over which specific outcome is reached; it is controlled only by the disturbance itself. It should be noted that having such a degree of variety does not necessarily say whether that regulator's reponses successfully counter every disturbance to yield an outcome in E, though it is more likely to do so.

In order to guarantee a favorable outcome, over a given epoch a system must (as a minimum requirement) have greater than or equal to the amount of variety of its environment, or $V_{system} \geq V_{environment}$. [3]

For the systems engineer, says Boardman, this may be interpreted as "for a system to survive in its environment the variety of choice that the system is able to make must equal or exceed the variety of influences that the environment can impose on the system." [8] In an environment of infinite possibilities, this implies that a system must have an infinite variety of choices in order to ensure success. It is obviously not feasible to design a system with infinite degrees of freedom, but it is possible to create systems that may succeed in an infinite environment for a finite period of time. Life itself is the clearest example. However, per the Law of Requisite Variety, a favorable outcome can not be *guaranteed*, but the more variety the more likely a system is to survive.

The concept of variety enables a smooth transition between multiple concepts, including entropy, complexity, and functionality. Entropy can be considered as a probabilistic measure of variety, and if every state is equiprobable, entropy reduces

to variety exactly. [30]

If entropy, H, expresses the uncertainty about a system's state, then if the state of a system is known exactly, $H = 0$ (state probability $= 1$). Using this approach, the LRV may be formulated in terms of entropy, as Ashby demonstrated in Equation 7. In this equation, E is the set of essential (desired) system states, D is the set of environmental disturbances, R is the set of system responses, and K is a buffering term (reflecting dissipative factors in the total system). [30]

$$H(E) \geq H(D) - H(R) - K \tag{7}$$

To visualize each of the terms, recall that entropy is essentially a measure of the number of possible states for a system; therefore, $H(R)$ is a reflection of the number of possible states for the regulator. As Heylighen emphasises [30], however, the formulation in Equation 7 includes the assumption that the system will always perfectly select the best response to counter any particular disturbance. Allowing for imperfect selection yields a revised relation of equation 8. [30]

$$H(E) \geq H(D) + H(R|D) - H(R) - K \tag{8}$$

The conditional entropy term $H(R|D)$ represents the uncertainty that the response set can match the disturbance set. It is calculated as usual for entropy, but the conditional probabilities are used. If response r is one response in set R ($r \in R$) and disturbance d is a disturbance in set D ($d \in D$), then the conditional entropy for each disturbance is found using Equation 9. $H(R|D)$ is calculating by summing over the full set of D [30].

$$H(R|d) = -\sum_{r \in R} p_f(r|d)\ln(p_f(r|d)) \tag{9}$$

2.7 Summary

The literature review demonstrates that a wide variety of research areas bear upon this topic. The next two chapters will shape this research material into new examinations of functionality, complexity, and how they may be measured.

III. Functionality

Clark, Jason B., David R. Jacques, John M. Colombi, and William P. Baker. "Assessing system functionality using an energy-based approach". Journal of Systems Engineering (submitted article), 2015.

3.1 Introduction

Systems engineering is a study of trade-offs. It is trading high value for low cost; seeking high efficiency while possessing high flexibility; trying to appear unfathomable to unfriendly systems while transparent to cooperative ones; or attempting to attain complete situational awareness without paralysis by information overload.

Those trade-offs are generally characterized as trade-offs in functionality. However, despite its widespread usage, there does not appear to be a unified approach to the measurement of functionality. There is common agreement that a loss of functionality is "bad" and more functionality is "good", yet precise quantification of more or less functionality remains elusive. (The caveat to the argument of more functionality being good is that it is only good if the cost and schedule impacts are properly balanced.)

It is the aim of this paper to provide a method to practically assess the functionality of systems, or changes to systems' functionality, in order to design more capable and resilient architectures.

3.2 Definition of Systems

An expansive treatment by Kuras defined a system as "a collection of properties and relationships that is both a whole and a part of a more expansive whole." [43] This definition emphasizes the notion of how a system exists at multiple scales (as discussed below) and thus fits with the motivations of the research topic. There are some recent

attempts at redefinition of "system" to account for difficulty of engineering complex systems with multiple layers of interacting systems.

For the purposes of engineering, the definition of "system" typically includes reference to performance or function. The definition used by Maier and Rechtin for system is "a set of different elements so connected or related as to perform a unique function not performable by the elements alone." [46] This definition does not explicitly account for scale or environment. The International Council of Systems Engineering (INCOSE) adopted a similar definition. [36]

Both definitions are "correct", and both are necessary to develop the definition for dynamic systems used for this paper. This paper adopts the Kuras definition for system.

Definition 1 *A* System *is a collection of properties and relationships that exists as both a whole and a part of a more expansive whole.*

Definition 2 *A* Dynamic System *is a collection of properties and relationships which is capable of changing itself or the environment within which it exists in a manner not performable by the elements alone.*

The requirement for functionality in the definition of system drives treatment of only dynamic systems, which is the type of concern to engineering, rather than including some more abstract concepts.

3.3 Definition of Functionality

As mentioned earlier, there does not appear to be a unified approach to the measurement of functionality. A representative definition for function is "a task, action, or activity that must be accomplished to achieve a desired outcome or provide a desired capability." [42] While accurate, more precision is required before approaching

a measureable quantity.

Stryker [63] defines function as “[a] technical process involving energy, material, and/or signals being converted and or channeled.” The energy, material, or signals acted upon are defined as flows. This definition is used as the basis for the definition of function used in this paper:

Definition 3 *A* Function *is a technical process involving flows within a system that modifies the state vector (including quantity, direction, type, amplitude, frequency, or phase) of a system.*

Mathematically, a function, f_i, is a technical process within a system S that modifies a subset of the state vectors $X_i \subset X_S$. That is, $f_i \in F_S$ such that $f_i : \alpha \subset X_S \to \beta \subset X_S$ where α is a subset of the state vectors X_S achievable by S called the domain of f_i, and β is called the range of f_i.

A state vector refers to the state space representation, or mathematical model, of a physical system. The state (or state vector) of a system at a particular time is the information that, together with an input set, determines the output set. [14] The state vector includes all attributes of the system, such as degrees of freedom, constraints, system status (e.g. failure modes), and logical structure.

As stated earlier, flows could include energy, material, or signals, although this paper will focus primarily on changes in energy. Note that any process that modifies a state vector as defined will involve some change in the kinetic energy vector for the flow.

We note there is always the function $f_0 \in F_S$ (not necessarily unique) which has the property that for every $x \in X_S$, $f_0(x) = x$. This function will be called the “identity function”. In other words, it is always possible to define a function that maps a system state back onto itself. Further, we can define the composition of functions on X_S, denoted $\circ$, as $f_j \circ f_i : \alpha \subset X_S \to \gamma \subset X_S$ and $f_j \circ f_i \in F_S$.

With a precise measurable definition for function, it is now possible to proceed to defining functionality. It should be noted that there is an established body of work known as "Function Point Analysis", which is defined as measuring the business functionality that an information system provides to a user[38]. However, this field applies exclusively to software, as opposed to dynamic systems, and it applies only to estimating the cost, duration, and amount of resources required to develop a software project. This research seeks to establish and analyze functionality with a systems engineering view, which is a very different context than the business motivations of Function Point Analysis.

Stryker defines functionality as "the number of functions to be performed by a system." [63] The International Council on Systems Engineering (INCOSE) defines functionality as "a set of attributes that bear on the existence of a set of functions and their specified properties". [35] While these definitions are agreeable, this paper effectively combines these definitions into one which does not tie functionality to measurement of functions, which can be arbitrarily partitioned.

Buede [11] defines functionality as "a set of functions that is required to produce a particular output". In addition, Simple Functionality is defined as "an *ordered* [sic] sequence of functional proceses that operates on a single input to produce a single output. Also, Complete Functionality is defined as "a complete set of coordinated processes that operate on all of the necessary inputs for producing a specific output".

Definition 4 Functionality *is the ability of a system, through use of functions, to affect its degrees of freedom.*

Mathematically, for α and $\beta \subset X_S$, functionality, $F_{\alpha,\beta}$, is the minimal set of all distinct functions $f \in F_S$ such that $f(\alpha) = \beta$. It should be noted that there may be multiple paths (multiple distinct functions, f) to move from one state to another.

Degrees of freedom refers to the number of generalized coordinates that define a

system's allowable motion in configuration space. The concept is distinct from the state vector in that degrees of freedom are dictated by the dimension of the embedding phase space for the system. The degrees of freedom for a system may be constrained below the state dimension for a given system.

To illustrate the idea of functionality as defined, if a surveillance aircraft loses its ability to elevate its camera turret, it has lost both functionality (the ability to affect its internal states related to elevating the camera) and a degree of freedom (a parameter describing the turret angle). Additionally, some functions and degrees of freedom are only available with certain state values. For example, an aircraft is unable to rise vertically unless state variables such as speed, aerodynamic surface positions, and other variables are within certain value ranges.

There may be ordered pairs $(\alpha, \beta) \in X_S \times X_S$ for which $F_{\alpha,\beta}$ is empty; this means the system can not transition from state α to state β. In other words, a system may not possess the functionality to reach all states in the system. However, if the system is acted upon by the environment, that functionality could be made to exist.

This definition for functionality allows a range of options for its measurement. This paper will take the approach of measuring the energy flows of a system, which will be developed in a subsequent section.

3.4 Systems and Functionality

As mentioned, a system exists at multiple scales. This paper will adopt (with small clarifications) the definition of system scale by Kuras [43]:

Definition 5 *The* System Scale *is a conceptualization (characterized by the information content) of a system due to a particular focus. This focus is realized through either a combination of a particular field-of-view and resolution of observation or a particular pattern.*

The concept of field-of-view is well-understood in the field of systems architecture. Changing the observer's perspective of the system yields new insights. Linked together, multiple views portray a more complete understanding of the system. [46]

Patterns are a rich topic of discussion, but are in essence a structure (conceptualization) of a system as perceived by an observer. A key point here is that the focus (and system scale) can enable the perception of a different pattern of a system. [43] For illustration, consider the dramatic change in perspective when focusing on one pattern or the other in depth-perception-based holograms.

These concepts lead to one of the fundamental concepts in the functionality metric developed in this paper, that of a functional level.

Definition 6 *A* Functional Level *is the system scale above which a particular functionality is realized.*

Referring back to the definition of functionality, this can also be read as "the system scale above which a system's ability to affect particular degrees of freedom is realized." The concept of Functional Levels is illustrated in Fig. 3 (adapted from Sheard and Mostashari [60]), where the functionality of a system can be decomposed into progressively finer scales. Note that the functionality of interest may include a particular degree of freedom or a set of them. Specifying a particular functional level allows a consistency of detail across systems, allowing comparative analysis.

An example assignment of types of functional levels is shown in Table 1. In this instance, Missions, Mission Tasks, and Functions might be assigned to Levels 1, 2, and 3, respectively. For an Unmanned Aircraft Systems (UAS), a typical mission might be to support humanitarian operations. To help perform that mission a UAS would need to conduct surveillance of an area of interest in order to monitor for threats, in addition to other tasks such as selecting landing zones. To support the task of surveillance, a UAS would need to first locate a threat, then characterize, identify,

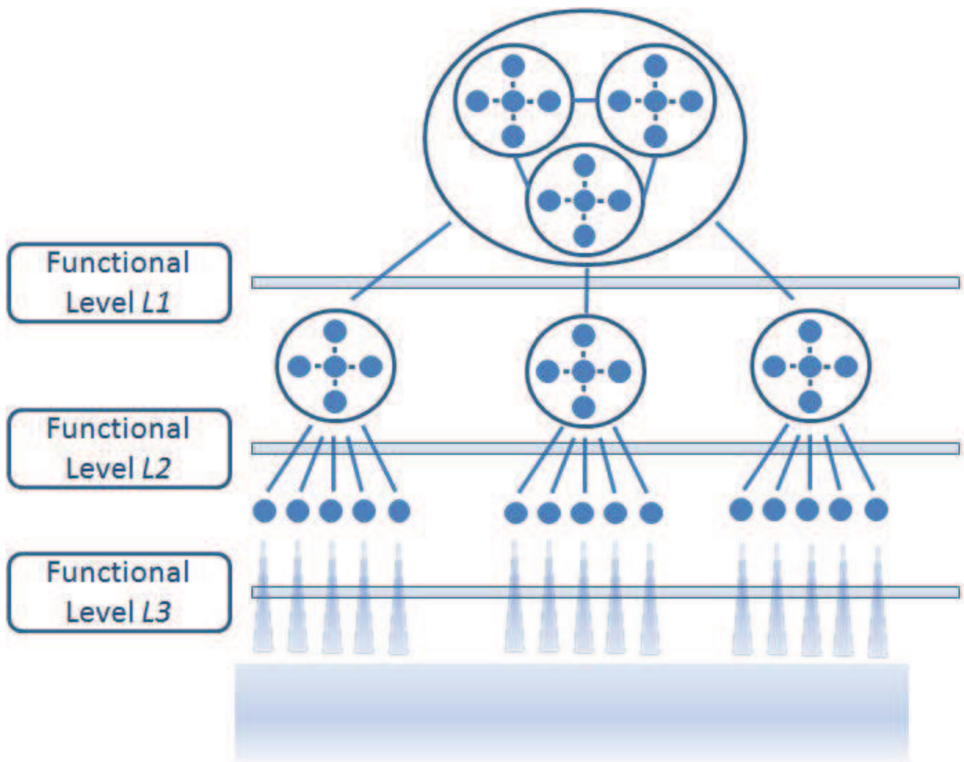

Figure 3. Functional Levels for a notional system viewed at multiple scales

Table 1. **Examples of Functional Levels**

Functional Level	Common Types	UAS Example
L1	Mission	Support Humanitarian Operations
L2	Mission Tasks	Conduct Area Surveillance
L3	Functions	Locate Threat

geo-locate, and track the target (if mobile).

3.5 Measuring Functionality

The preceding discussions set the stage for establishing a metric of functionality for dynamic systems. Dynamic systems, by definition, yield changes in their kinetic energy over time. Using this fact, we can define a Functional Interaction:

Definition 7 *A* Functional Interaction *is an instance where, for a discrete element of a system, its kinetic energy in the direction of a particular degree of freedom transitions from a non-zero value to a zero value, or from a zero to a non-zero value.*

For a mechanical system, kinetic energy is represented as: $KE_d = \frac{1}{2}m|\vec{v}\cdot\vec{d}|^2$ where KE_d is kinetic energy in the $\vec{d}$ direction, m is the system mass, and $\vec{v}$ is the velocity vector. A functional interaction occurs when that value reaches zero or becomes greater than zero. For a car travelling along a road, a functional interaction occurs when the car stops, starts again, or turns.

One additional definition is required, that of scenario.

Definition 8 *A* Scenario *is a unique set of system conditions to include inputs, constraints, initial state, and events.*

The preceding discussion allows for the following definition of Realized Functionality over an epoch, an arbitrary unit of time used to measure an event:

Definition 9 Realized Functionality *($\mathcal{F}$) of a system is the number of functional interactions for a given scenario within a particular epoch and above a particular functional level.*

This is, in essence, functionality by demonstration. A static system may have potential for functionality, but its functionality cannot be measured until it is active. Depending on the scenario, a system may respond with differing amounts of functionality. A plane flying through clear skies may stay straight and level, but be required to make complex maneuvers when flying through a storm.

Realized Functionality aligns with Buede's Simple Functionality discussed earlier. As he describes, Simple Functionality may not include all of the necessary functional processes needed to produce the output under all scenarios, nor does it trace the only possible sequence of those processes. [11] Similarly, Realized Functionality focuses on a specific scenario. It differs from Simple Functionaliy in that Realized Functionality does not focus on the inputs, but rather the flow of functions (or interactions) through a system.

A key to the utility of this definition is the ease with which it may be applied. For instance, if we choose an appropriate epoch, an aircraft's ability to fly is dependent upon exercising a finite number of functional interactions. That is, it must ensure a constant flow of energy throughout various systems, and perform work using that energy, in order to survive. Aero surfaces must move, engine components must operate, and multiple other subsystems must be conducting functional interactions. If the realized functionality is reduced, it means that part of the aircraft system is processing less energy. If functionality at a certain level of abstraction is reduced, the system is either reduced in function by mission (e.g. steady level flight) or by damage (e.g. inoperable engine, aero surfaces).

Note that system failures may appear to violate the above assertion with respect to processing energy. In the case of an engine fire, the system may appear to be processing energy at a higher rate while functionality is reduced (less useful thrust is produced). However, in failures such as these a functional or destructive limit has been reached, a concept which will be addressed in a later section.

Important to this definition is "functional level". For instance, a human's functionality may be measured at the cellular, molecular, or atomic levels, and given the same epoch size, the $\mathcal{F}$ value increases for each progressively finer system scale.

There are other ways we can illustrate the functionality of systems. For example, a single-path system can have the same $\mathcal{F}$ value as a multi-path network (Fig. 20). The main difference between these systems is the path for the first system is predictable, whereas the network path is dependent on additional factors whose properties must be known to make predictions. It is the difference between the *maximum realizable functionality* and the *expected functionality* of the two systems.

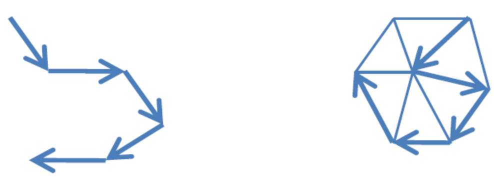

Figure 4. Two systems with identical realized functionality. (a) Single-path (b) Multi-path Network

Definition 10 Maximum Functionality, *$\mathcal{F}_M(\sigma, L, \tau, \Psi)$, is the the maximum realizable functionality for a system (σ) at a functional level L over an epoch τ, for any given scenario S_i over the range of all possible scenarios Ψ, as in Equation 10.*

$$\mathcal{F}_M(\sigma, L, \tau, \Psi) = sup(\mathcal{F}(\sigma, L, \tau, \Psi)|S_i), S_i \in \Psi \tag{10}$$

Realized functionality may be used to measure maximum functionality. If it was possible to observe a system's response to all possible scenarios and measure the functionality, this information could be used to construct a description of the maximum functionaltiy of the system. As the certainty of a system's structure and functionality decrease, the certainty in (and ability to measure) maximum functionality is also reduced.

Realized functionality, as defined earlier, is a single observation of system responding to a particular scenario. *Expected Functionality* for a system σ may thus be written as Equation 13 where F is the Cumulative Density Function (CDF) of Ψ.

$$\mathcal{F}_X(\sigma, L, \tau, \Psi) \equiv E[\mathcal{F}(\sigma, L, \tau, \Psi)] = \int_{\Psi} \mathcal{F}(\sigma, L, \tau, S_i) dF(S_i) \geq 0 \tag{11}$$

For a set of finite, discrete, and independent scenarios, expected functionality for a system σ may be written as Equation 14 where $p(S_i)$ is the probability mass function over Ψ.

$$\mathcal{F}_X(\sigma, L, \tau, \Psi) = \sum_{\Psi} \mathcal{F}(\sigma, L, \tau, S_i) p(S_i) \geq 0 \tag{12}$$

For a given system, one scenario may cause it to respond with a particular behavior, whereas another may produce a completely different behavior. As defined here, scenarios which produce identical behavior in the system(s) under study are considered identical (they cannot be differentiated even though they can be distinguished). This enables a framework for system study where the infinite possibilities of system scenarios may be parsed to a workable, finite set. For most systems of interest, this finite set may still be too large for practical analysis, but the solution to this problem will be addressed subsequently.

Just as Realized Functionality aligns with Buede's Simple Functionality, Expected Functionality aligns with his Complete Functionality. Both definitions encompass a

complete description of system's functionality across all scenarios. As with Realized and Simple Functionality, Expected and Complete Functionality differ in that Expected Functionality focuses on expected functional flow through the system rather than inputs and outputs.

3.5.1 Agents and Functionality.

Agent-based models are illustrative of many concepts in complex systems, and measures of functionality should reflect increasing functionality with increasing activity of agents.

"Boids" (Bird-androids) are a type of agent based model using rule sets based on the motions of animal flocks, herds, and swarms. [15] Compare a 3-Boid flock against 3 equivalent non-interacting aircraft. In going from point A to point B, which takes place over a particular epoch in both cases, non-interacting aircraft will make flight path adjustments due to atmospheric disturbances. A Boid-aircraft will make these adjustments as well, but also make adjustments due to each of the Boid guidance rules due to aircraft position/velocity co-dependencies. At first glance, it might appear that the Boid flock has higher functionality than the set of non-interacting aircraft.

Since non-cooperative flight is a subset of cooperative flight (equivalent to setting certain rule weightings to zero), if no Boid rules are invoked, both systems have equivalent functionality. However, a Boid rule might lead to behavior requiring fewer flight corrections by, for example, causing all aircraft to align quickly and reduce the need for evasive maneuvers. Thus, predictions of the functionality of agent-based systems can not be made for any particular rule-set. The functionality is highly dependent on the vehicle states and capabilities, as well as the environment. Making specific predictions of functionality based on a rule set is a difficult proposition.

3.5.2 System Example: Unmanned Air System (UAS).

In order to illustrate the application of realized functionality, consider an UAS with a gimbaled camera designed to image a target. For the purposes of this example there are four degrees of freedom (DoF): x, y, ϕ, and θ, where x and y are Cartesian position coordinates, ϕ is the camera turret elevation, and θ is turret azimuth. An image may be obtained by adjusting the state space $\mathbf{x} =[x\ y\ \phi\ \theta]^T$ through a control vector $\mathbf{u}$.

First, consider the case where all degrees of freedom are enabled: $\mathbf{u}= [a(t)\ b(t)\ c(t)\ d(t)]^T$, where a, b, c, and d are functions driving a constant rate of motion in each DoF from start to stop. Here, the system may be defined as $\sigma : \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$. If each DoF is exercised by the control vector, $\mathcal{F}(\sigma, L, \tau, \Psi) = 8$ (by virtue of only one start and stop per DoF). This sets a lower bound on $\mathcal{F}_M$.

In the next case, the same UAS is restricted to move in only 3-DoF, having lost the ability to rotate the turret due to damage (or a change to a fixed camera). The vector $\mathbf{u} = [a(t)\ b(t)\ c(t)\ 0]^T$ reflects the loss of control. If each available DoF is exercised by $\mathbf{u}$, $\mathcal{F} = 6$, a lower bound for $\mathcal{F}_M$ of this new system. This result is intuitive, since the loss of a degree of freedom should reflect a loss in the functionality of the system. The same trend is evident if we restrict the motion still further, such that $\mathbf{u} = [0\ 0\ c(t)\ 0]^T$. In this case, $\mathcal{F} = 2$.

Extending the UAS example still further, suppose that the full-authority system is able to satisfy the image equation by adjusting the value of only two DoF's. However, the controller opts to make movements in all 4 DoFs to obtain an image. This results in an "excess" functionality, behavior which is not necessary to reach the goal, even if it may allow for a shorter required time duration to obtain an image.

This example demonstrates why it is important to keep in mind the goals and limitations of a functionality measure. Functionality is a reflection of a system's

potential behaviors, not of efficient behaviors. The choice of control vector, given a system's functionality, is what controls efficiency. Irrational actors can implement control policies that produce highly complex behaviors, yet yield no benefit.

The effectiveness of a system can be measured by the choice of control vector against some objective function. That effectiveness, however, is not a measure of its functionality but what it does with that functionality.

In this UAS example, expected functionality may be calculated by determining a probability mass function for the scenario set and summing the product of that function with the realized functionality for each scenario (as in Equation 14). Practically, the scenario set will consist of the most likely operating conditions. The measure of expected functionality may then be used to compare systems and determine suitability. Similarly, maximum functionality (as in Equation 10) provides additional insight into the functionality potential of the system, but expected functionality gives a more complete picture for guiding system design.

3.6 Functionality, Energy Limits, and Energy Available

At first glance, it appears the $\mathcal{F}$ measure increases without bound as energy increases. However, recall that $\mathcal{F}$ is based on physical processes and is subject to physical limits; there is a limit to the amount of energy that physical systems may contain (e.g. fuel tanks, vehicle structural limits).

The functional energy limits for a system depend on the form of the energy, the rate of energy addition, and system structure.

Definition 11 *A* Functional Limit *is the maximum amount of a type of energy a system or a component can acquire before the system performs a function (to limit energy gain) or reaches its capacity to absorb energy. The functional limit may be dependent on the rate of energy addition for some systems, in that the functional limit*

can increase over time provided that energy is added at a sufficiently low rate.

A functional limit for an aircraft fuel tank is reached when the tank is at capacity. The aircraft could also take action to halt fuel intake (perform a function) prior to reaching capacity. In contrast, a destructive limit is reached if the tank is overpressurized and bursts. In reference to the rate of energy addition, this is not meant to imply merely replace energy used for the same total limit. For example, a flexible fuel bladder may have a larger capacity if it is filled at a slow rate than at a fast rate, which would cause it to burst sooner.

Definition 12 Energy Available *to a system is energy that is in a form where the system can use it directly.*

Energy is only usable to a system in particular forms. Consider solar energy: it is not directly available to a typical aircraft, and therefore cannot be considered available energy to the aircraft system. The addition of a photovoltaic cell to the system, however, can allow direct use of solar energy. The solar energy could also be indirectly used by soaring on thermal updrafts, for example.

A system may avoid reaching its destructive energy limit for a particular form of energy by imposing a functional limit. Using the example above, a photovoltaic/-battery system typically has a self-limiting mechanism that prevents consumption of energy from reaching the destructive limit: overloading of the battery. However, such a system has no functional mechanism to prevent reaching the destructive limit if energy is in the form of intense electromagnetic energy (e.g. a high-powered laser).

These limits, functional and destructive, are dependent on both the form of energy and the system under study. It should also be noted that a functional limit mechanism may have an upper saturation limit which can be overcome with high enough energy. For example, a munition may be engineered to be insensitive to electromagnetic

radiation (i.e. absorb it), a functional mechanism which might be defeated with high enough energy levels of the same type of radiation.

The preceding definition enables the development of an assertion describing the general relationship between energy and functionality.

If all other parameters are held constant, as the energy available to a system increases, the system will maintain or increase in expected functionality, provided that the destructive energy limit for the system is not exceeded. The converse is also true: as the energy available to a system decreases, the system will maintain or decrease in expected functionality.

So, as stated, as a system absorbs energy, functionality increases until reaching a functional or destructive energy limit. If a functional limit, functionality will remain constant. If a destructive limit, functionality will fall.

Using this assertion, it can be said that the addition of energy can move $X_S(\alpha)$ in time. The time evolution of the system state vector, and even the system size (the minimal set of attributes for a complete system characterization), are influenced by effects from the environment such as energy. The addition or removal of information may also have such an effect, which will be explored in future research. Furthermore, as a system uses energy (dissipating it to the environment), $X_S(\alpha)$ shrinks. There are fewer states to reach and less functionality to reach other states.

3.7 System Robustness and Resiliency

While it is generally asserted that increased functionality is a positive factor, this section serves to illustrate how expected functionality directly contributes to system success. Functionality is directly linked to success through the concept of reachable states; specifically, states associated with success and the probability of reaching them.

Engineered systems are designed in response to a capability gap, or in order to perform a set of tasks. For any engineered system in a particular scenario set, there will be a set of final states which represent the desired outcome. These states may be referred to as the success states.

As discussed in the previous section, the system state vector, $X_S(\alpha)$, for a system can change size in response to changes in energy (in particular, this means that existing states could become frozen or enabled in response to energy changes). This means that there is a change in the set of states that are reachable. Recall that system state at a particular time includes all system attributes such as degrees of freedom, constraints, system status, and logical structure. In general, a system with higher functionality has more states available (reachable) than one with lower functionality.

While higher functionality can be associated with increased resiliency and robustness, the concepts are not synonymous. As defined by the INCOSE Reslient Systems Working Group, "Resilience is the capability of a system with specific characteristics before, during and after a disruption to absorb the disruption, recover to an acceptable level of performance, and sustain that level for an acceptable period of time." [37]

Robustness is the ability of a system to reject disturbances without altering its state. A system is robust when it can continue functioning in the presence of internal and external challenges without fundamental changes to the original system [35]. In relation to the previous section on energy availability, robustness is the abiliy for a system to retain reachable states in the event of falling available energy.

For this paper, robustness will be defined as the ability to remain in a mission capable state and resilience will be defined as the ability to return to a mission capable state. A mission capable state is one from which it is possible to reach a state that defines success.

The earlier section answered the question of what happens to functionality when the energy available changes. This section will use resiliency and robustness to answer the question of why excess functionality is desirable, again using the example of an Unmanned Air System. In short, functionality that may be redundant in achieving reachable success states may allow them to remain reachable in the face of degradation (such as reduced available energy).

3.7.1 Functionality and Resiliency: A UAS Example.

Returning to the UAS with a gimbaled camera introduced earlier, suppose its objective is to collect high-quality images of a particular target set. The success states therefore include only outcomes where the UAS is positioned in such a manner that it is within a specific range of the target, the target is within the camera's field of view, and the slant range is less that the maximum allowable. Clearly, any outcomes where the UAS does not come within range of the target, whether due to energy availability, threats, or damage constitute failure states. A state machine diagram for the UAS is illustrated in Figure 9.

The UAS has a set of "mission capable" states that represent those states from which the success states are achievable without a modification of the system. In other words, the mission capable states are those where a function exists to move from that state to a success state. Mathematically, if the set of success states is defined as $B \subset X_S$ and the set of mission capable states is $A \subset X_S$, for any $\alpha \in A$ there exists a $\beta \in B$ and an $F_{\alpha,\beta}$ such that an $F_{\alpha,\beta} = \beta$.

In contrast, the failed states represent those from which the success states are not achievable. Mathematically, for all $\alpha \in A$ there does not exist a $\beta \in B$ where $F_{\alpha,\beta} = \beta$.

Suppose the UAS suffers damage or degradation as before where it has lost the

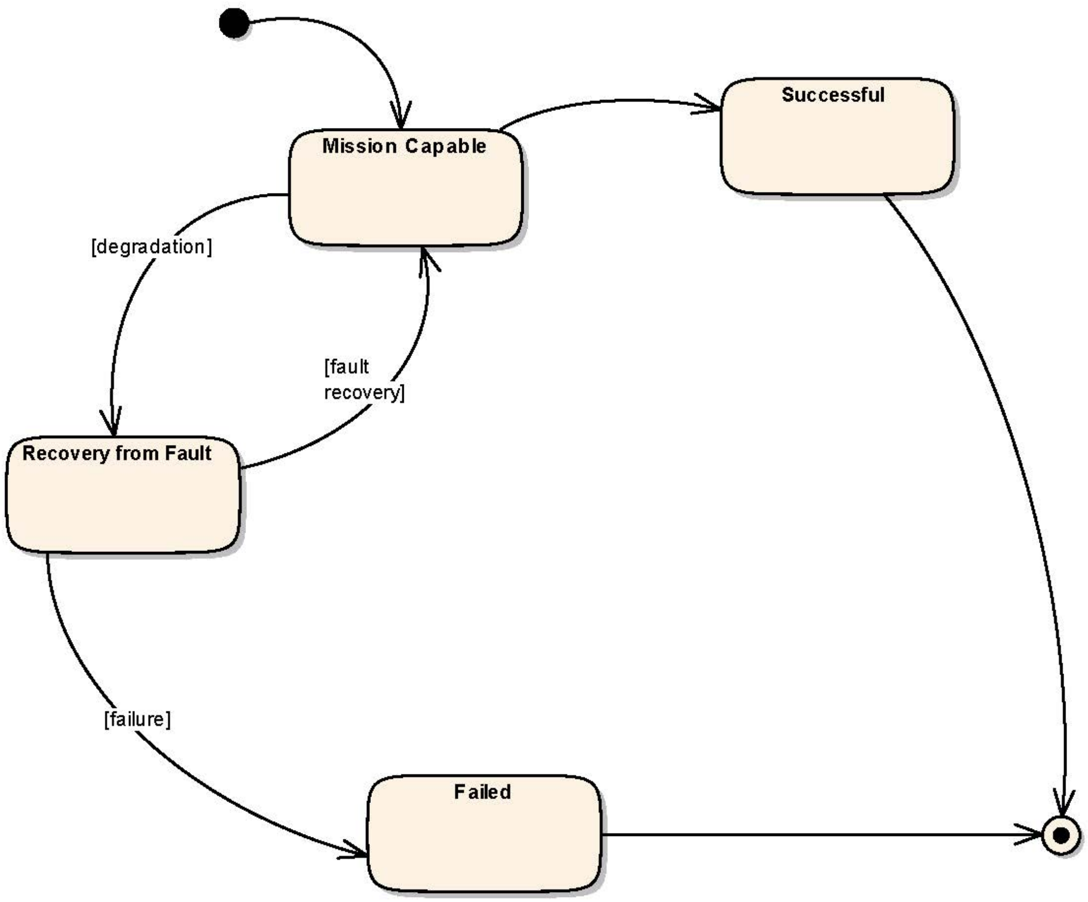

Figure 5. State machine diagram for UAS operations

ability to rotate the turret. At this point, the UAS enters a state where it attempts to recover from the fault, from which the success states are not achievable without system modification. To return to the set of mission capable states, the UAS must adjust its control laws in order to position the camera without the turret rotation DoF. This involves a manipulation of the other DoF's, such as following a different flight path.

The probability of success, $P_{success}$, for the UAS against its objective can be calculated if the transition probabilities for the mission capable and recovery from fault states are known.

3.7.2 Adaptability.

Life forms and civilizations fall into a class of complex systems that are often termed "adaptive". While definitions vary, a system that changes itself in response to disturbances from the environment is generally considered as adaptable. The change may manifest itself in structure or behavior, although presumably a change in structure leads to a change in behavior.

In contrast, a flexible system can withstand or be modified in response to current or expected disturbances from the environment (this is related to the definition for robustness earlier). Flexibility here is adapted from Ryan's definition of Design Flexibility, which is defined as the degree to which a system can be modified to increase its capability in response to forseeable external change [55].

Adaptation often seems to imply an improved suitability (fitness) to the environment, but the definition used here will not include such value judgements. Fitness is determined according to some function (such as probability of survival) that could yield different results depending on timescale or range of possible low-probability high-impact (black swan) events. Thus, the ability to change (adapt) in response to

the environment may not lead to a statistically favorable change in the long-term.

For the purposes of this research, an adaptive sytem will be defined in relation to its expected functionality, which is a behavior-based metric.

Definition 13 *An* Adaptive System *is one that can vary its expected functionality over time.*

It should be noted that any practical observation of adaptive systems per this definition requires appropriate selection of τ, the epoch of observation.

Referring to the UAS discussion above, a system which is capable of traversing through the recovering state is by definition adaptable.

3.7.3 Decline and Collapse.

In traditional combat models, such as those based on Lanchester equations, combat is a process of attrition. The competitor with the superior combination of combat effectiveness and numerical strength is pre-destined to emerge the victor. As real combat, particularly the asymmetric conflicts of modern times, demonstrates, this model has limited utility for accurately predicting combat outcomes. Real combat is "nonlinear, uncertain, and complex with no outcome ever taken for granted." [61]

One distinction between the two concepts is that between decline and collapse. In traditional combat models, decline or degradation is the only form of relative performance measurement. In real combat, there is not only smooth decline in functionality, but rapid jumps that may be termed collapse.

As discussed above, graceful degradation or decline is manifest in a slow reduction in expected functionality and collapse is a rapid reduction in expected functionality.

For this research, collapse will be defined as follows:

Definition 14 Collapse *is a rapid reduction in expected functionality, such that* $0 > \epsilon > \frac{d\mathcal{F}_X}{dt}$*, where* ϵ *is an arbitrary constant.*

In contrast, decline is defined as:

Definition 15 Decline *is a slow reduction of expected functionality, such that* $0 > \frac{d\mathcal{F}_X}{dt} > \epsilon$*, where* ϵ *is an arbitrary constant.*

A future goal of this research is to find values of ϵ for given systems, determine the factors necessary for transitioning between decline and collapse (or alternatively, increasing functionality), and if there are any patterns useful for understanding engineered systems in general.

The value of ϵ will assuredly be dependent on the scale of the system. For example, the ϵ for modern civilization might be very small as compared to a military system. The speed of the reduction in functionality that might be a disaster for a civilization could be an easily acceptable loss in a battle. Putting bounds on ϵ that relate to system specifics could be very useful.

In practical terms, a decline (graceful degradation) allows a system to transition to the recovering state. A collapse only allows transition to the failed states.

3.8 Conclusion

This paper established a number of definitions derived from literature, which support a method for measurement of system functionality. The terms defined include system, dynamic system, function, functionality, system scale, functional level, and scenario. Collectively, these terms enable definitions for realized functionality and expected functionality, which use changes in kinetic energy to measure system functionality for a given scenario or over the set of all possible scenarios, respectively.

In summary, this paper demonstrates the validity and use of a new measure of

functionality in dynamic systems. Realized functionality can be used to compare systems in identical scenario sets, measure historical functionality, or set bounds on the maximum functionality. The work outlined in this paper meets the primary goal for this research: developing a method to measure the functionality of a system in a practical manner.

This paper also demonstrated clear linkages with assessing resilience and robustness for a system. Future research on this topic will correlate the probablility of system success with functionality, resilience, and robustness.

IV. Information, Functionality, and Complexity

4.1 Introduction

Accounting for the impact of information in the design of systems and their function is a continuing challenge for systems engineers. Of primary concern for this research is the effect of collecting information, processing it, and using it in order to improve the functionality of a system.

The United States Department of Defense has put significant emphasis on Intelligence, Surveillance, and Reconnaissance (ISR) systems dedicated to the collection, processing, and dissemination of information [39]. As ISR collection systems and environments evolve, the need for a clear understanding of the relative merits of competing system designs has become more evident. Some systems require extensive manpower to spend long hours analyzing video [53], others provide information in such volumes and formats that analysts experience information overload [40].

How much information do systems need in order to perform their missions? Does information drive or enable system functionality? What role do changes in information processes have in system failure or collapse? How can information overload be managed? These are all questions for which this paper provides the foundations for answers.

4.2 Functionality of Systems

Functionality and functions are fundamental concepts in designing systems [42]. Functions involve converting or channeling energy, material, and signals. An earlier work by the authors [18] explored relationships between functionality and energy (and by extension, material). This paper explores the relationship between functionality and information (i.e. signals).

Several key definitions are required from the authors' earlier work in order to set the context for this work. For a complete treatment, please refer to "Assessing system functionality using an energy-based approach" by Clark, et. al [18].

*A **Function** is a technical process involving flows within a system that modifies the state vector (including quantity, direction, type, amplitude, frequency, or phase) of a system.* Mathematically, a function, f_i, is a technical process within a system S that modifies a subset of the state vectors $X_i \subset X_S$. That is, $f_i \in F_S$ such that $f_i : \alpha \subset X_S \rightarrow \beta \subset X_S$ where α is a subset of the state vectors X_S achievable by S called the domain of f_i. β is the range of f_i, and also a subset of the state vector.

***Functionality** is the ability of a system, through use of functions, to affect its degrees of freedom.* Mathematically, for α and $\beta \subset X_S$, functionality, $F_{\alpha,\beta}$, is the minimal set of all distinct functions $f \in F_S$ such that $f(\alpha) = \beta$. It should be noted that there may be multiple paths (multiple distinct functions, f) to move from one state to another. This fact is of critical importance in understanding how information impacts the increase or decrease of functionality.

*A **System** is a collection of properties and relationships that exists as both a whole and a part of a more expansive whole.*

*A **Dynamic System** is a collection of properties and relationships capable of changing itself or the environment within which it exists in a manner not performable by the elements alone.*

*A **Functional Level** is the system scale above which a particular functionality is realized.* The functionality of a system can be decomposed into progressively finer scales. The functionality of interest may include a particular degree of freedom or a set of them.

*A **Functional Interaction** is an instance where, for a discrete element of a system, its kinetic energy in the direction of a particular degree of freedom transitions*

from a non-zero value to a zero value, or from a zero to a non-zero value. For a car travelling along a road, a functional interaction (with the road) occurs when the car stops, starts again, or turns.

*A **Scenario** is a unique set of system conditions to include inputs, constraints, and initial states.*

The preceding definitions allow for the definition of Realized Functionality over an epoch, an arbitrary unit of time:

*The **Realized Functionality**, $\mathcal{F}$, of a system is the number of functional interactions for a given scenario within a particular epoch and above a particular functional level.* Realized functionality is a measurement of demonstrated functionality.

4.2.1 Expected functionality.

Realized functionality, as defined earlier, is a single observation of a system responding to a particular scenario. *Expected Functionality* for a system σ may thus be written as Equation 13 where $\mathcal{F}_X$ is the expected functionality for a system (σ) at a functional level L over an epoch τ, for any given scenario S_i over the range of all possible scenarios Ψ. F is the Cumulative Density Function (CDF) of Ψ.

$$\begin{gathered}\mathcal{F}_X(\sigma, L, \tau, \Psi) \equiv E[\mathcal{F}(\sigma, L, \tau, \Psi)] = \\ \int_\Psi \mathcal{F}(\sigma, L, \tau, S_i) dF(S_i) \geq 0\end{gathered} \tag{13}$$

For a set of finite, discrete, and independent scenarios, expected functionality for a system σ may be written as Equation 14 where $p(S_i)$ is the probability mass function over Ψ.

$$\mathcal{F}_X(\sigma, L, \tau, \Psi) = \sum_\Psi \mathcal{F}(\sigma, L, \tau, S_i) p(S_i) \geq 0 \tag{14}$$

4.2.2 Expected functionality and energy available.

Previous work by the authors demonstrated analytically that a system's expected functionality is driven by the energy available to that system. [18] Energy available to a system is energy that is in a form where the system can use it directly. However, there are limits to the amount of energy a system may absorb, either destructive or imposed by the system.

This framework enables the following assertion: *If all other parameters are held constant, as the energy available to a system increases, the system will maintain or increase in expected functionality, provided that the destructive energy limit for the system is not exceeded. The converse is also true: as the energy available to a system decreases, the system will maintain or decrease in expected functionality.*

In other words, increasing energy increases functionality in a system until that system stops the input or breaks.

4.3 Functionality and complexity

In systems engineering, the concepts of functionality and complexity are often intertwined. Complexity creeps in as the desire for increased functionality required for solutions to daunting challenges grows. Its characteristics appear in suddenly critical problems of modern civilization that were nonexistent mere years before. And of primary interest for this paper, complexity provides the needed link between functionality and information effects.

As this research has examined and precisely defined functionality, it is now necessary to do the same for complexity. This presents a challenge, however, as there are many (often mutually exclusive) definitions for complexity. The following section will present these multiple views of complexity, but for clarity the key ideas on complexity asserted in this paper are stated here: 1) Complexity is measurable for any dynamic

system; 2) Complexity is positively correlated with functionality; and 3) Complexity is manifest through the behavior of a system. These ideas will be developed in this section.

4.3.1 Definitions of complexity.

There are a wide variety of approaches to defining and measuring complexity throughout the literature on complexity theory. In many cases throughout the literature, it seems the authors are speaking of different or even mutually exclusive phenomena. The most apt summary of the diversity of approaches is the tongue-in-cheek assertion of the nature of complexity by researcher Seth Lloyd: “I can’t define it for you, but I know it when I see it” [20].

A root cause in the lack of unified definitions of complexity is that there are in fact several types of complexity. The first formal treatment of complexity focused on algorithmic complexity, which reflects the computation requirements for a mathematical process. [20] Senge and Sterman include also dynamic complexity, which is primarily characterized by difficult-to-discern cause-effect relations [59] [62]. To further muddy the waters, there are the concepts of Complex Systems and Complex Adaptive Systems.

Sheard and Mostashari [60] discuss some aspects that describe a complex system. These include: being composed of many autonomous, yet interacting, components or agents (leading to such phenomena as feedback); being self-organizing; displaying emergent macro-level behavior; and adapting to the environment (fitness landscape).

Complex Adaptive Systems (CAS) extend the concept of complex systems into a more specialized class. A CAS can be characterized by the following properties: the system consists of a network of many agents acting in parallel; it has many levels of organization; it anticipates the future; and it has many fitness “niches” which can

be exploited by agents adapted for the purpose. [67] All of these properties serve to enable emergence – a collective property of a system that cannot be predicted by examining each agent individually.

Important to CAS is the concept of dynamic equilibrium, or homeostasis. CAS are “living” systems (usually not just figuratively), with constantly moving processes. [48] With a dynamic system, something must always occur (or be occurring) for the system to operate. If that something is interrupted or modified it can have profound effects on the system’s seemingly stable behavior.

These concepts (complex systems and CAS) help define a class of systems that may be considered “complex”. The definition of this class is useful for understanding when to apply particular analysis and design techniques. In order to apply these techniques, it is necessary to understand what comprises complexity and whether it is a measurable aspect.

One of the most workable definitions of complexity is that of thermodynamic depth. Properties of thermodynamic depth as complexity assert that complexity “is a measure of how hard it is to put something together”. [45] [22] [44]

Bar-Yam defines the complexity of a physical system as the length of the shortest string, s, that can represent its properties. This can be the result of measurements and observations over time. [5] However, there are some inherent difficulties in using these definitions. For one, it is very difficult to approach a macro-level system, particularly one with hidden structure, and determine its information content. While information and thermodynamics-based measures form a theoretical basis for complexity measurement, there remains a need for a more accessible route; particularly one relevant for systems engineering.

There are hints that energy plays a role in defining complexity. As a general rule, systems commonly recognized as complex process more energy than less complex

systems. For instance, civilizations move process energy than a single town. However, energy flow is not a good measure of complexity, since a single arbitrary large-bore flow, such as large "pipe" (a relatively simple system), could be constructed with the identical energy flow as a more complex system.

A metric that solves the apparent large-bore contradiction is proposed by Chaisson, which was initially used as a means to classify the complexity of stars. By measuring the energy rate density in Equation 47, where Φ_m is the energy rate density, E is energy flow through a system, τ is the time epoch, and m is system mass, Chaisson obtains results that correlate well with other notions of complexity. [13] However, the energy rate density metric has some drawbacks. By normalizing with respect to mass, this metric produces incorrect results for complexity when comparing some systems. For example, suppose an electronic brain is built to mimic the operation of a human brain. The human brain may process energy at the same rate as a theoretical electronic brain, but due to differences in basic materials (i.e. the weight of neurons vs. semi-conductors), the two systems, which most theorists would recognize as identically complex, could have vastly different Φ_m values. Thus, by normalizing with respect to mass instead of function, the Φ_m metric produces incorrect results for the relative complexity of systems.

$$\Phi_m = \frac{E}{\tau m} \tag{15}$$

A practical difficulty in using the Φ_m metric is determining the appropriate mass and energy to use. In measuring the Φ_m of a civilization, Chaisson uses the mass of humanity and the total energy processed by the civilization. However, the total energy of a civilization does not flow through only its humans, but also its machinery, beasts of burden, vehicles, etc., the mass of which is a difficult quantity to measure.

4.3.2 Behavioral complexity and functionality.

All complexity metrics discussed thus far contribute ideas toward a measure that:

- Correlates with notions of complexity at all scales
- Is measurable for any dynamical system
- May be practically employed

Further development is needed in order to define a measure of complexity that meets these criteria and is thus useful for systems engineering. To bridge that gap, the authors propose a new approach to viewing complexity.

As discussed in the previous section, complexity is described as a measure of computational difficulty, difficulty of assembly, or difficulty of description. As stated previously, the authors view these as different aspects of the same phenomenon. In addition to the three aspects just described, it is proposed that the complexity of a system can be illustrated by observing its behavior – the assertion here is that a system that is difficult to compute, assemble, or describe will behave in a manner that is difficult to compute, assemble, or describe. If one can measure this behavior in a meaningful way, it should provide a reliable barometer of the system's inherent complexity.

This notion of complex behavior being a characteristic of complex systems is not necessarily new. Consider the energy rate density metric: Chaisson's core assertion is that increasingly complex systems will process energy at a (mass-normalized) higher rate. This is a clear link between system complexity and behavior. The Φ_m metric, however, has drawbacks for its wider application as discussed earlier.

The measure of behavioral complexity is the measure of how a system functions. As such, it is proposed that expected functionality can serve as a proxy measure of a system's behavioral complexity. The authors assert here that system functionality

increases with system complexity. That is, a more complex system will have a higher expected functionality. The next section will explore the validity of this relationship.

4.4 Functionality and information

4.4.1 Linking functionality with information-based complexity.

The purpose of this paper is to explore the effect of changing information on functionality. To enable this exploration, it is necessary to first link functionality with complexity.

Previous research has established complexity metrics through use of information theory. Bar-Yam and others define complexity of a system as the length of the shortest information string s that can completely represent the properties of that system. This complexity value is equivalent to the information content of a system [5].

This paper does not propose that expected functionality is equivalent to Bar-Yam complexity, but it is asserted here that there should be a correlation between the two measurement approaches. This assertion is difficult to demonstrate analytically, however it is possible to demonstrate common trends with limited examples. It is important to consider that realized functionality measures system behavior whereas Bar-Yam complexity measure system information content. As discussed, it follows intuitively that a system with more complex behavior would take more information to describe than a system with less complex behavior.

Consider a vehicle-road system such that the vehicle is constrained to move according to a grid as shown in Figure 6. The car starts any given scenario at point ‘A’, may travel along the line segments (each of length N), but may only start or stop at the labeled locations (e.g. no stopping in the middle of the freeway).

The scenario set, Ψ, is the set of all possible scenarios that drive a unique system response. For a given time epoch, $\tau = 6T$ in this case, the complexity of each scenario

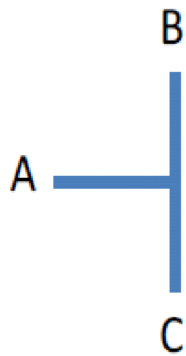

Figure 6. Vehicle and road system 1

is shown in Table 2. Examining scenario S_2 more closely, the complexity is calculated by first counting the starts and stops in particular directions: start at A, stop at intersection; start in new direction, stop at B. This yields a total of 4 counts. As the epoch size increases or the car gains functionality (e.g. greater maximum speed), complexity increases.

Bar-Yam complexity, description of the system, requires an information coding scheme for measurement. Since the road network itself is static, its information entropy $H = 0$. To describe the behaviour of the car, one must know τ; whether it moved from A, or whether it moved to B or C; from there, whether it moved again; and so forth.

Assume that when the car moves, it does so at a speed of 1 N/T, where T is the time unit. An example of the minimum string length encoding $\tau = 5$T is shown in Figure 7.

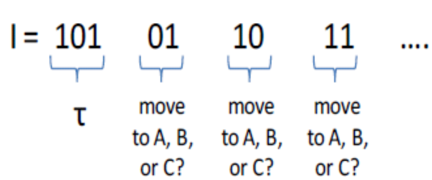

Figure 7. Information string describing the movement of the vehicle

Table 2. Behavioral complexity of car given particular scenarios

Scenario	Functionality	Distance	Order
S_1 = Car doesn't move	$\mathcal{F} = 0$	0L	1
S_2 = Move from A - B	$\mathcal{F} = 4$	2L	2
S_{2A} = Move from A - B - C	$\mathcal{F} = 6$	4L	3
S_{2A1} = Move from A - B - C - A	$\mathcal{F} = 10$	6L	4
S_{2A2} = Move from A - B - C - B	$\mathcal{F} = 8$	6L	5
S_{2B} = Move from A - B - A	$\mathcal{F} = 8$	4L	6
S_{2B1} = Move from A - B - A - B	$\mathcal{F} = 12$	6L	7
S_{2B2} = Move from A - B - A - C	$\mathcal{F} = 12$	6L	8
$\vdots$	$\vdots$	$\vdots$	
S_3 = Move from A - C	$\mathcal{F} = 4$	2L	9
S_{3A} = Move from A - C - B	$\mathcal{F} = 6$	4L	10
S_{3A1} = Move from A - C - B - A	$\mathcal{F} = 10$	6L	11
S_{3A2} = Move from A - C - B - C	$\mathcal{F} = 8$	6L	12
S_{3B} = Move from A - C - A	$\mathcal{F} = 8$	4L	13
S_{3B1} = Move from A - C - A - B	$\mathcal{F} = 12$	6L	14
S_{3B2} = Move from A - C - A - C	$\mathcal{F} = 12$	6L	15
$\vdots$	$\vdots$	$\vdots$	

For τ = 2T, we obtain the following set of encodings for each possible scenario: I1 = 10 00; I2 = 10 01; I3 = 10 10. The first block is the time, 2, converted to binary and the next block designates 00 for staying at A, 01 for moving to B, or 10 for moving to C. The set Ψ = S1 S2 S3; no other set of scenarios can cause the system to respond in a unique way in the given epoch. The expected functionality, assuming a uniform distribution of scenarios, is calculated as in equation 16.

$$\mathcal{F}_X = 0 * \tfrac{1}{3} + 4 * \tfrac{1}{3} + 4 * \tfrac{1}{3} \approx 2.67 \tag{16}$$

By varying the epoch size, vehicle speed, and size of the road network, a series of values for expected functionality and string length (s) can be calculated. The results for varying scenario distributions are shown in Figure 8. The Gaussian Normal distribution is defined according to equation 17, where μ is the mean (in this case the

mean scenario) and σ is the standard deviation. For Figure 8, $\mu = 8$ and $\sigma = 2$. The Triangular distribution is calculated according to equation 18 where a is the lower limit, b is the upper limit, and c is the mode. For Figure 8, a, b, and c vary with the size of the scenario space to (arbitrarily) correspond with the first, middle, and last on the list, respectively. The scenarios are ordered according to the last column of Table 2.

$$P(x) = \tfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 \big/ 2\sigma^2} \tag{17}$$

$$P(x) = \begin{cases} 0, & \text{for } x < a \\ \dfrac{2(x-a)}{(b-a)(c-a)}, & \text{for } a \leq x < c \\ \dfrac{2}{b-a}, & \text{for } x = c \\ \dfrac{2(b-x)}{(b-a)(b-c)}, & \text{for } c < x \leq b \\ 0, & \text{for } b < x \end{cases} \tag{18}$$

Using the results from this example, it is reasonable to state that expected functionality and Bar-Yam complexity have an analogous relationship. While a closed proof of this relationship has not been completed, this assertion is sufficient to proceed with using techniques established forcomplexity on functionality.

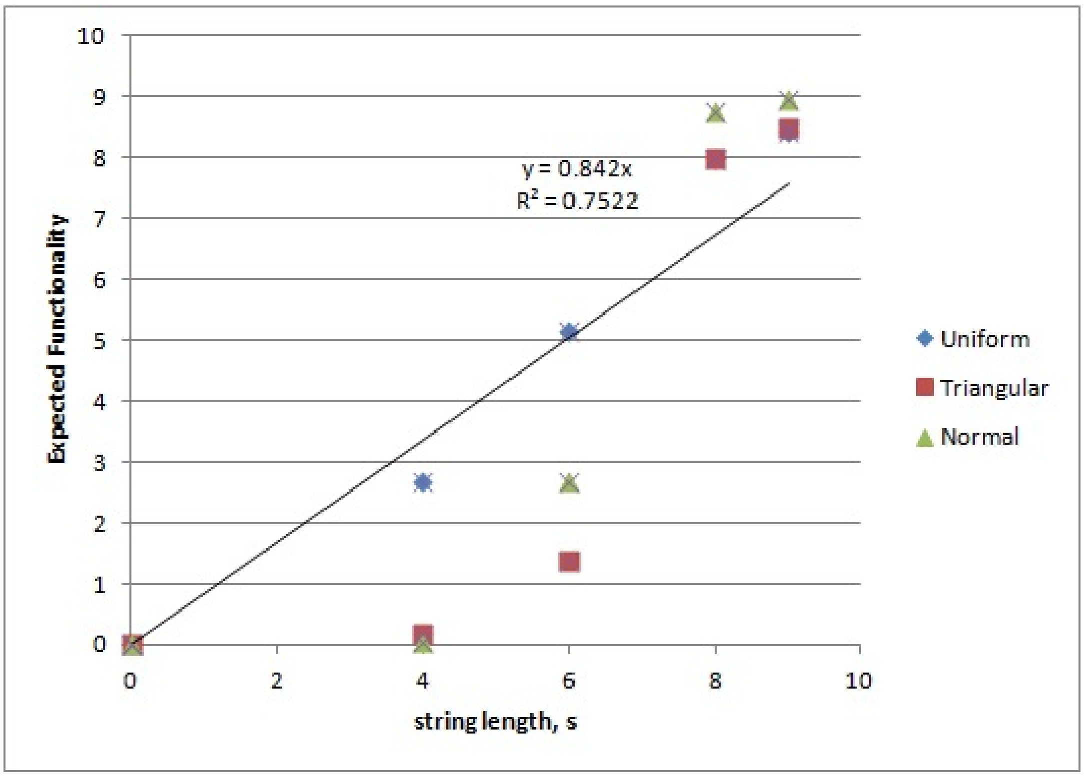

Figure 8. Shortest string length vs. Total complexity for the vehicle-road example system for different scenario set distribution types

4.5 Effect of information on expected functionality

As stated earlier, the purpose of this paper is to establish the relationship between information and functionality in the same manner already accomplished for energy and functionality. To do so requires exploration of key concepts in information theory.

4.5.1 Requisite variety.

Ashby's Law of Requisite Variety (LRV) defines variety as the number of elements (or states) in a set that can be distinguished. [3] As an example, the set {b r b a b r} has a variety of 3 letters. Variety can refer to specific events, scenarios, or actions. In order for a regulator R to control a set of disturbances D to yield a specific favorable outcome from set E (the set of all desired outcomes) it must have at least as many possible responses as there are disturbances. [3] If the same response was applied to two different disturbances, there would be two different values in the outcome set, E (a specific outcome is determined by both the disturbance and response). This is uncontrolled regulation, since there is no control over which specific outcome is reached; it is controlled only by the disturbance itself. It should be noted that having such a degree of variety does not necessarily say whether that regulator's reponses successfully counter every disturbance to yield an outcome in E, though it is more likely to do so.

In order to guarantee a favorable outcome, over a given epoch a system must (as a minimum requirement) have greater than or equal to the amount of variety of its environment, or $V_{system} \geq V_{environment}$. [3]

For the systems engineer, says Boardman, this may be interpreted as "for a system to survive in its environment the variety of choice that the system is able to make must equal or exceed the variety of influences that the environment can impose on the system." [8] In an environment of infinite possibilities, this implies that a system

must have an infinite variety of choices (degrees of freedom) in order to ensure success. It is obviously not feasible to design a system with infinite degrees of freedom, but it is possible to create systems that may succeed in an infinite environment for a finite period of time. Life itself is the clearest example. However, per the Law of Requisite Variety, a favorable outcome can not be *guaranteed*, but the more variety the more likely a system is to survive.

The concept of variety enables a smooth transition between multiple concepts, including entropy, complexity, and functionality. Entropy can be considered as a probabilistic measure of variety, and if every state is equiprobable, entropy reduces to variety exactly. [30]

In information theory, a dynamic system (Ω, B, P, Z) is measured using a finite alphabet mapping, $f : \Omega \rightarrow A$, where Ω is a measurable event space, B is a particular collection of subsets of Ω, P is a probability measure, Z is a transformation of the event space such that $Z : \Omega \rightarrow \Omega$ (such as time), and A is an alphabet.[27] It should be noted that an alphabet is not limited to something akin to our familiar A through Z; a set as vast as the complete set of words in the English language can constitute an alphabet in this treatment.

The entropy of a random variable, a, from that alphabet may be defined as:

$$H(f) = -\sum_{a \in A} p_f(a) \ln(p_f(a)) \tag{19}$$

where p_f is the probability mass function of f. [27] This expression is clearly analogous to thermodynamic entropy. The difference is the thermodynamic states, designated here as Ω, have been mapped to a set of information states, A.

Just as thermodynamic entropy is regarded as a measure of the uncertainty of the state of a system, information entropy is considered a measure of the uncertainty of the information about the state of a system. Using this understanding of the nature

of information uncertainty, it is possible to understand the links between variety, complexity, and information.

If entropy, H, expresses the uncertainty about a system's state, then if the state of a system is known exactly, $H = 0$ (state probability = 1). Using this approach, the LRV may be formulated in terms of entropy, as Ashby demonstrated in Equation 20. In this equation, E is the set of essential (desired) system states, D is the set of environmental disturbances, R is the set of system responses, and K is a buffering term (reflecting dissipative factors in the total system). [3]

$$H(E) \geq H(D) - H(R) - K \tag{20}$$

To visualize each of the terms, recall that entropy is essentially a measure of the number of possible states for a system; therefore, $H(R)$ is a reflection of the number of possible states for the regulator. As Heylighen emphasises [30], however, the formulation in Equation 20 includes the assumption that the system will always perfectly select the best response to counter any particular disturbance. Allowing for imperfect selection yields a revised relation of equation 21.

$$H(E) \geq H(D) + H(R|D) - H(R) - K \tag{21}$$

The conditional entropy term $H(R|D)$ represents the uncertainty that the response set can match the disturbance set. It is calculated as usual for entropy, but the conditional probabilities are used. If response r is one response in set R ($r \in R$) and disturbance d is a disturbance in set D ($d \in D$), then the conditional entropy for each disturbance is found using Equation 22. $H(R|D)$ is calculated by summing over the full set of D. [30]

$$H(R|d) = -\sum_{r \in R} p_f(r|d) \ln(p_f(r|d)) \tag{22}$$

Bar-Yam demonstrated the extension of variety to his measurement of complexity [6] as shown in equation 23, where $P_{success}$ is the probability of a desired outcome, $C(e)$ is the complexity of the environment, and $C(a)$ is the complexity of the system in question. Recall that Bar-Yam complexity is measured using binary string length, hence the base 2 logarithm. Note that the buffering term, K, is neglected. Also, Bar-Yam's equation makes the earlier assumption of perfect application of responses.

$$0 \leq -\log_2(P_{success}(a)) < C(e) - C(a) \tag{23}$$

As was discussed earlier, expected functionality may be assumed analogous to information complexity for the purposes of this paper. It is then possible to rewrite equation 23 as equation 24, where Ψ_e and Ψ_a are the scenario distributions for observing the environment (e) and system (a), respectively.

$$-\log_2(P_{success}(a)) < \mathcal{F}_X(e, L, \tau, \Psi_e) - \mathcal{F}_X(a, L, \tau, \Psi_a) \tag{24}$$

In order to incorporate "imperfect" systems, we must include a term that functions as the $H(R|D)$ term in equation 21. If we define the probability that the system behaviour response does not match (counter) the environmental disturbance as $P(Y_a)$, then the term defined in equation 25 captures the lack of ability for a system to perfectly counter its environment. In contrast to $\mathcal{F}_X$, $\mathcal{F}_Y$ is the expected functionality conditioned on the occurrence of all the events Y_a where the system reponse does not counter the disturbance. We may then rewrite equation 24 as equation 26.

$$\mathcal{F}_Y(a, L, \tau, \Psi_a) \equiv \sum_{\Psi_a} \mathcal{F}(a, L, \tau, S_i|Y_a)\ P(Y_a)\ p(S_i) \tag{25}$$

$$
\begin{gathered}
-\log_2(P_{success}(a)) < \\
\mathcal{F}_X(e, L, \tau, \Psi_e) + \mathcal{F}_Y(a, L, \tau, \Psi_a) - \mathcal{F}_X(a, L, \tau, \Psi_a)
\end{gathered} \tag{26}
$$

Another factor impacts success probability: the amount of information a system has about its environment, or the observed complexity (functionality). Up until now, it was assumed that the system had perfect knowledge of its environment and so will choose the best response. Under most practical scenarios, it is not possible to know all the information amount the environment or another system, and therefore the true value of this ratio cannot be known. However, comparisons between systems using this framework are useful, as will be shown in later sections. The term I_e is defined here as the ratio of information the system possesses about the environment to the total information that exists about the environment. This is conveniently expressed using ratios of entropy, such that $I_e = 1 - H_o(e)/H(e) = (H(e) - H_o(e))/H(e)$ where $H(e)$ is the maximum possible information entropy for the environment and $H_o(e)$ is the information entropy after observation of the environment. If the environment is completely known by the system, $I_e = 1$ since the uncertainty in observation is zero. Conversely, if the system possesses no information on the environment, $H_o(e) = H(e)$ and $I_e = 0$. The inclusion of this consideration yields equation 27.

$$
\begin{gathered}
-\log_2(P_{success}(a)) < \mathcal{F}_X(e, L, \tau, \Psi_e) + \\
\mathcal{F}_Y(a, L, \tau, \Psi_a)\frac{1}{I_e} - \mathcal{F}_X(a, L, \tau, \Psi_a) I_e
\end{gathered} \tag{27}
$$

In the complete absence of information, $I_e = 0$ (and its inverse goes to infinity), and $P_{success} \geq 0$. Under perfect information, $I_e = 1$, and the expression reverts to the previous one (equation 26). Note that this is a different concept than the imperfect knowledge term; that term assumes perfect observation of the environment. If the system can't observe its environment, it can't possibly know which response to apply.

4.5.2 Effect of information changes.

In the authors' earlier work [18], it was shown how the addition of energy can move $X_S(\alpha)$ in time. Specifically, the addition of energy (below destructive limits) increased the size of $X_S(\alpha)$ and the removal of energy decreased it. Similarly, this section will demonstrate that the addition of information reduces the likelihood of a decrease of $X_S(\alpha)$, provided that both the cost of obtaining the information is relatively small and the information can be processed by the system.

Clearly, a winning strategy for all systems is to maximize the amount of information known about the environment. Unfortunately, there is an energy cost to obtaining information. For every unit of information obtained, it costs the gaining system some amount of energy to gather it and process it, which in turn has the potential to reduce functionality. This relates to the problem of information overload or paralysis. Tainter [66] describes this as the problem that as knowledge grows more complex, the production of that knowledge becomes subject to diminishing returns. Stated in terms of the definitions established in this paper, as the functionality of the system under observation increases, the cost to process information about that system increases.

This concern can be resolved with the addition of a correction term that accounts for the energy cost of information, $\phi(E_A, E_C(I_e, \Delta I_e))$. Here, ϕ is a function of the energy available, E_A, and energy cost of the change in information, $E_C(I_e, \Delta I_e)$, such that $\phi(E_A, 0) = 1$ and $\phi(E_A, E_A) = 0$. Notionally, ϕ approximates as in equation 28, although the exact form for it may differ for any given system. The function ϕ will not graduate as smoothly for all systems, but the form given in equation 28 can be considered a reasonable approximation (this follows from the Taylor approximation). (Note that E_C has units of energy.)

$$\phi(E_A, E_C(I_e, \Delta I_e)) \approx 1 - \frac{E_C(I_e, \Delta I_e)}{E_A} \tag{28}$$

Incorporating the correction term ϕ leads to a revised expression for $P_{success}$ from equation 27 to that in equation 29.

$$\begin{gathered} -\log_2(P_{success}(a)) < \mathcal{F}_X(e, L, \tau, \Psi_e) + \\ \mathcal{F}_Y(a, L, \tau, \Psi_a) \frac{1}{I_e + \Delta I_e} \frac{1}{\phi(E_A, E_C(I_e, \Delta I_e))} - \\ \mathcal{F}_X(a, L, \tau, \Psi_a)(I_e + \Delta I_e)\phi(E_A, E_C(I_e, \Delta I_e)) \end{gathered} \tag{29}$$

From this expression, it can be shown that $P_{success}$ increases with increased information only if $\phi(E_A, E_C(I_e, \Delta I_e)) > 0$. Stated simply, the probability of success for a system increases with the addition of information, as long as the cost of obtaining that information is less than the energy available. The smaller the cost, the more impact the information has on increasing the probability of success. A further condition, however, is that the energy cost is not sufficient to reduce functionality; recall that a reduction in energy available results in lower or an unchanged value for expected functionality. In other words, the probability of success for a system increases with the addition of information, provided $E_C << E_A$.

The discussion above also allows for the following assertion regarding functionality (as opposed to probability of success above):

As a system's information on its environment increases, the probability that a system will apply the most suitable functions increases, provided that $E_C << E_A$.

From Clark 2015 [18], it is known that as energy available, E_A, increases the size of $X_S(\alpha)$ increases. Simliarly, as E_A falls, $X_S(\alpha)$ decreases. Since adding more information (ΔI_e) to a system reduces E_A by an amount E_C, the addition of that

information causes a decrease in the size of $X_S(\alpha)$. However, if a system posesses more information, it is more likely to select the correct function from within $F_{\alpha,\beta}$, the minimal set of all distinct functions $f \in F_S$ such that $f(\alpha) = \beta$. The lack of that information does not remove any inherent functionality, but it reduces the system's probability of successfully applying the most suitable functions.

From this discussion, it can be said that information increases the "functional efficiency" of a system. Thus, as stated before, the more information a system gains, the greater its chances for success. Less information about the functionality of the environment increases the probability of encountering a destructive or functional limit. Encountering a destructive limit reduces functionality, which reduces chances of success and moves the system into a "Recovery from Fault" state (see Figure 9, which is reprinted from [18]. The probability of transition to the success states is lower since there is a non-zero probability of transition to the set of failed states.

A system has a set of "mission capable" states from which the success states are achievable without a modification of the system. In other words, the mission capable states are those where a function exists to move from that state to a success state. Mathematically, let the set of success states be defined as $B \subset X_S$ and the set of mission capable states be $A \subset X_S$, then given $\alpha \in A$, there exists a function $f \in F_{\alpha,\beta}$ such that $f(\alpha) = \beta$.

In contrast, the failed states represent those from which the success states are not achievable. Mathematically, let C be the set of all "Failed" states, then for all $c \in C$ and all $\beta \in B$, $F_{c,\beta} = \emptyset$ (the empty set).

To illustrate, a car that lacks information about an on-rushing car is more likely to encounter a destructive limit than a car possessing that information. That information is not enough, however; the car must also possess a high-enough functionality (i.e. ability to maneuver or brake) to have a chance of escaping its fate. This illustrates

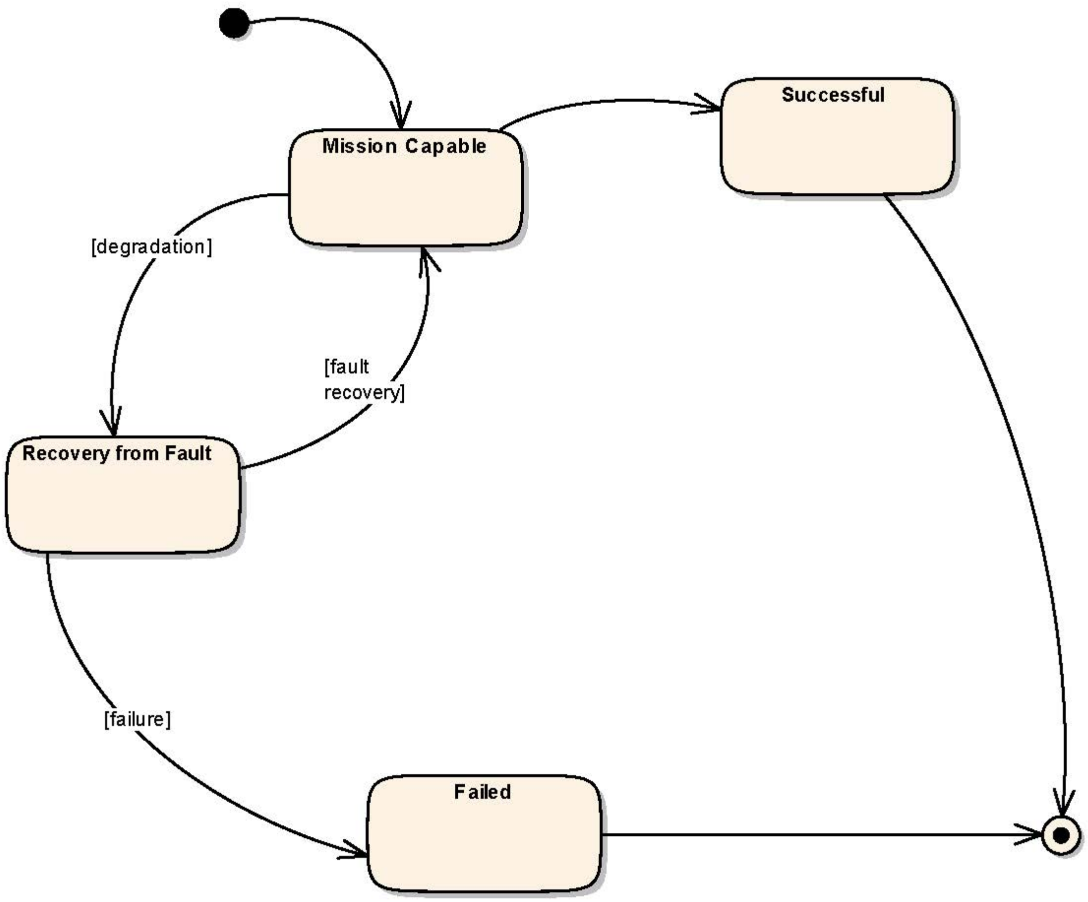

Figure 9. State machine diagram for UAS operations

a compounding feedback mechanism inherent to dynamic systems: decreased information leads to a system decreasing its functionality due to poorly applied functions, and both effects further yield a decrease in probabilty of success.

Stated in this way, information is a preventative factor. It helps prevent reduction of functionality, but does not directly increase functionality. This idea also accounts for irrelevant information. If information is obtained about a completely unrelated system, such as the behavior of an anti-aircraft system in a distant country, it has no bearing on improving the chances of maintaining or increasing the functionality of a combat UAS. In fact, the act of obtaining that information could negatively impact probability of survival, as shown above.

4.6 Using expected functionality for system applications

4.6.1 Cooperative systems.

There are a number of applications to which these relations can be applied. One possible application is cooperative systems, such as Boids [15]. Here, systems $a1$ and $a2$ refer to two boid-governed systems (e.g. aircraft) that make up a cooperative flock. It is assumed for simplicity that systems $a1$ and $a2$ have the same observations of the external environment, yielding equations 30 and 31.

$$\begin{gathered} -\log_2(P_{success}(a1)) < \mathcal{F}_X(e, L, \tau, \Psi_e) + \\ \mathcal{F}_Y(a1, L, \tau, \Psi_{a1})\frac{1}{I_e} - \mathcal{F}_X(a1, L, \tau, \Psi_{a1}) I_e \end{gathered} \tag{30}$$

$$\begin{gathered} -\log_2(P_{success}(a2)) < \mathcal{F}_X(e, L, \tau, \Psi_e) + \\ \mathcal{F}_Y(a2, L, \tau, \Psi_{a2})\frac{1}{I_e} - \mathcal{F}_X(a2, L, \tau, \Psi_{a2}) I_e \end{gathered} \tag{31}$$

In the case of cooperative systems, success of one system is positively related to that of the other. Adding Equations 30 and 31 captures the reinforcing effect of cooperation on success probability:

$$
\begin{aligned}
-\log_2(P_{success}(a1)) - \log_2(P_{success}(a2)) < \\
\mathcal{F}_X(e, L, \tau, \Psi_e) + \mathcal{F}_Y(a1, L, \tau, \Psi_{a1})\frac{1}{I_e} \\
-\mathcal{F}_X(a1, L, \tau, \Psi_{a1})I_e + \mathcal{F}_X(e, L, \tau, \Psi_e)+ \\
\mathcal{F}_Y(a2, L, \tau, \Psi_{a2})\frac{1}{I_e} - \mathcal{F}_X(a2, L, \tau, \Psi_{a2})I_e
\end{aligned}
\tag{32}
$$

Alternately, this may be written as:

$$
\begin{aligned}
-\log_2(P_{success}(a1)\ P_{success}(a2)) < \\
2 * \mathcal{F}_X(e, L, \tau, \Psi_e)+ \\
(\mathcal{F}_Y(a1, L, \tau, \Psi_{a1}) + \mathcal{F}_Y(a2, L, \tau, \Psi_{a2})\frac{1}{I_e} \\
-(\mathcal{F}_X(a1, L, \tau, \Psi_{a1}) + \mathcal{F}_X(a2, L, \tau, \Psi_{a2}))I_e
\end{aligned}
\tag{33}
$$

However, the expected functionality of each boid is dependent on the influence of its partner. For the system $a1$, its expected functionality can be decomposed into the portion that may be influenced by system $a2$ and the portion that may not. If we define the probability that system $a2$ may influence $a1$ as $P(W_{21})$, then the term defined in equation 34 captures only the functionality due to the influence of the cooperative system. In contrast to $\mathcal{F}_X$, $\mathcal{F}_{W_2}$ is the expected functionality conditioned

upon all the events where system $a2$ influences system $a1$. It is assumed that the probability of cooperative influence is independent of the scenario distribution. This limiting assumption can be relaxed by replacing $P(W_{21})$ with $P(W_{21}|S_i)$.

$$\mathcal{F}_{W_2}(a1, L, \tau, \Psi_{a1}) \equiv \sum_{\Psi_{a1}} \mathcal{F}(a1, L, \tau, S_i|W_{21})P(W_{21})p(S_i) \tag{34}$$

Similarly, the effect of boid system $a1$ on system $a2$ can be written as equation 35.

$$\mathcal{F}_{W_1}(a2, L, \tau, \Psi_{a2}) \equiv \sum_{\Psi_{a2}} \mathcal{F}(a2, L, \tau, S_i|W_{12})P(W_{12})p(S_i) \tag{35}$$

The complement, the probability that $a2$ may not influence $a1$, is defined as $P(W_{21}^c) \equiv 1 - P(W_{21})$. Equations 36 and 37 capture the functionality of $a1$ and $a2$, respectively, where the systems can not influence one another.

$$\mathcal{F}_{W_2^c}(a1, L, \tau, \Psi_{a1}) \equiv \sum_{\Psi_{a1}} \mathcal{F}(a1, L, \tau, S_i|W_{21}^c)P(W_{21}^c)p(S_i) \tag{36}$$

$$\mathcal{F}_{W_1^c}(a2, L, \tau, \Psi_{a2}) \equiv \sum_{\Psi_{a2}} \mathcal{F}(a2, L, \tau, S_i|W_{12}^c)P(W_{12}^c)p(S_i) \tag{37}$$

The influence of boid $a2$ on boid $a1$ (and vice versa) can be decomposed into influence resulting from information exchange (i.e. communication) and influence with no information exchange (i.e. lift from wingtip vortices, collision). The term I_{12} is defined here as the ratio of information system $a1$ possesses about $a2$ to the total information that exists about $a2$. On the other hand, mechanisms where $a2$ can influence $a1$ without the requirement of information exchange can be assumed as part of the environmental behavior. For systems with large separations between cooperative systems, this effect can be neglected entirely. Note the distinction here is that while it may be possible to infer information about the other system through

indirect sensing, collisions, or wingtip vortices, there is no directed transmission of information. Direct observation, such as optical detection, is considered directed transmission of information (via reflected or emitted photons).

The expected functionality for system $a1$ is therefore defined in equation 38.

$$\begin{aligned} \mathcal{F}_X(a1, L, \tau, \Psi_{a1}) \equiv \mathcal{F}_{W_2}(a1, L, \tau, \Psi_{a1}) I_{12} \\ + \mathcal{F}_{W_2^c}(a1, L, \tau, \Psi_{a1}) \end{aligned} \tag{38}$$

If $a2$ can not influence $a1$, $\mathcal{F}_X(a1, L, \tau, \Psi_{a1}) = \mathcal{F}_{W_2^c}(a1, L, \tau, \Psi_{a1})$. If $a1$ has no information on $a2$, $\mathcal{F}_X(a1, L, \tau, \Psi_{a1}) = \mathcal{F}_{W_2^c}(a1, L, \tau, \Psi_{a1})$.

For an n-boid system, the expected functionality for a given boid may be written as equation 39

$$\begin{aligned} \mathcal{F}_X(ai, L, \tau, \Psi_{ai}) = \sum_{\substack{j=1 \\ j \neq i}}^{n} \Big(\mathcal{F}_{W_j}(ai, L, \tau, \Psi_{ai}) I_{ij} \\ + \mathcal{F}_{W_j^c}(ai, L, \tau, \Psi_{ai}) \Big) \end{aligned} \tag{39}$$

For the n-boid system a, if mission success for the System of Systems is dependent on the success of each boid, the probability of success for the boid System of Systems as a whole may be defined as equation 40.

$$P_{success}(a) \equiv \prod_{i=1}^{n} P_{success}(ai) \tag{40}$$

If mission success does not require that every boid be successful, then the probability of success for the System of Systems becomes equation 41, adapted from reliability engineering [1]. For example, swarm success may be defined as at least one boid reaching a target area; if the rest are destroyed along the way, their lack of success does not negate the overall mission success. This section will build from equation 40 for clarity.

$$P_{success}(a) \equiv \sum_{i=k}^{n} \binom{n}{k} (P_{success}(ai))^i (1 - P_{success})^{n-i} \tag{41}$$

Based on equation 40, the general expression for $P_{success}(a)$ without cooperative influence may then be written as equation 42.

$$\begin{aligned} -\log_2(P_{success}(a)) < \prod_{i=1}^{n} \Big[&\mathcal{F}_X(e, L, \tau, \Psi_e) + \\ &\mathcal{F}_Y(ai, L, \tau, \Psi_{ai}) \frac{1}{I_e} - \mathcal{F}_X(ai, L, \tau, \Psi_{ai}) I_e \Big] \end{aligned} \tag{42}$$

Upon including cooperative influence equation 42 becomes equation 43.

$$\begin{aligned} -\log_2(P_{success}(a)) < \prod_{i=1}^{n} \Big[&\mathcal{F}_X(e, L, \tau, \Psi_e) + \\ &\mathcal{F}_Y(ai, L, \tau, \Psi_{ai}) \frac{1}{I_e} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \big(\mathcal{F}_{W_j}(ai, L, \tau, \Psi_{ai})(I_{ij}) \\ &+ \mathcal{F}_{W_j^c}(ai, L, \tau, \Psi_{ai}) \big) \Big] \end{aligned} \tag{43}$$

So, probability of success for a boid flock depends on the relative functionality of the boid system with respect to its environment, the information the boid system has about the functionality of its environment, and the information each boid has about the rest of the flock. The term I_{ji} can be regarded as the degree to which a system's component is aware of its other components. If a system doesn't know its capabilities, it is unlikely to use that unknown capability.

Since a system with high functionality has more direction changes (functional interactions) than a system with low functionality, it can be inferred that a higher functionality system is generally more difficult to track. For a boid being tracked by its neighbor, higher functionality results in a lower probability of success for the

tracker; if being tracked by an enemy, higher functionality results in a higher success rate for the boid. Thus, there is an inherent design tension with competing goals to keep the functionality for each individual Boid as small as possible, while maximizing the functionality of the Boid flock as a whole.

This result is consistent with theories from systems architecture (and in fact extends them), articulated by Maier and Rechtin with a heuristic: "In partitioning, choose the elements so that they are as independent as possible; that is, choose elements with low external complexity and high internal complexity." [46] The heuristic, as written, applies to cooperative systems. Thus, a boid aircraft should have high functionality at the subsystem level, but low functionality at the system level – so that its compatriots can track it. However, for competitive systems, the heuristic is reversed; the system should be as functional as possible. So, for a cooperative boid system engaged with an outside competitor: each boid should have high subsystem-level functionality, low system-level functionality, and high system-of-systems-level functionality. This extended heuristic emphasizes why engineering cooperative combat systems is truly a difficult design problem.

It is clear from this formulation that a flock with more information is more likely to be successful due to appropriate choice of functions (functional efficiency). This information could include details about the operating environment, such as winds, obstacles, and targets; and details about other boids, such as operating limits, predicted responses to disturbances, and so forth.

The effect of the heuristic set on functionality is more complicated to identify, since the effect depends on the nature of the rules. For instance, in a system with only Boid collision avoidance rules, the velocity matching rule tends to decrease realized functionality. Aligning the headings of the vehicles (one of the generalized coordinate directions) reduces the number of instances where their headings and relative positions

require change to avoid collision. The velocity matching rule would also likely reduce the number of obstacle collision course corrections required.

Determining the effect of neighborhood size on functionality is very difficult to predict, since the number of boids within another boid's neighborhood changes with time. It could be that this effect is highly system-dependent

4.6.2 Competitive systems.

The idea regarding the amount of information a system has on its environment can be generalized still further. If a competitive system can be isolated from the environment (e.g. the on-coming car in the earlier example), the survival probability expression for a system, designated $a1$ here, may be written as:

$$\begin{aligned} -\log_2(P_{success}(a1)) < \mathcal{F}_X(a2, L, \tau, \Psi_{a2}) + \\ \mathcal{F}_Y(a1, L, \tau, \Psi_{a1})\frac{1}{I_{a2}} - \mathcal{F}_X(a1, L, \tau, \Psi_{a1}) I_{a2} \end{aligned} \tag{44}$$

Success in this instance refers to success against a particular external system ($a2$). This equation, however, makes the assumption that the opposing system ($a2$) blindly applies disturbances. We need also consider the effects of an active opponent system, where the probability of success of its mission is:

$$\begin{aligned} -\log_2(P_{success}(a2)) < \mathcal{F}_X(a1, L, \tau, \Psi_{a1}) + \\ \mathcal{F}_Y(a2, L, \tau, \Psi_{a2})\frac{1}{I_{a1}} - \mathcal{F}_X(a2, L, \tau, \Psi_{a2}) I_{a1} \end{aligned} \tag{45}$$

Note that each system's application of responses (or disturbances, depending on your perspective) may still yield success even if blindly applied. However, the more information possessed by system $a1$, the more likely it is to be successful and conversely

for system $a2$.

One reason these relationships exist as probabilistic expressions, rather than deterministic, is that it is certainly possible for a low functionality system to defeat a high functionality system. Examples abound, such as a rock thrown into a jet engine, Spartacus defeating the Romans, and bacteria killing a human. Yet, over the full scenario space, a fighter jet is more likely to smite a rock thrower, the Romans eventually defeated Spartacus, and most bacteria or viruses are overcome by immune systems or medicines. The important point here is that higher functionality does not always mean victory, but improves the chances of seeing it.

4.6.3 Application Example: UAS System-of-Systems.

The following example serves to illustrate the application of the concepts developed thus far, particularly with cooperative systems. Suppose a System-of-Systems (SoS) composed of 10 small UAS (Unmanned Air Systems) is tasked to observe activity at a fixed SAM (Surface-to-Air Missile) site. The success criteria for the SoS is defined as: Continuous coverage of the target in the interval between first arrival on station and end of task. "Coverage" occurs when a UAS is within 200 ft.

Suppose that the cooperative algorithm under consideration allows the UAS to influence one another only when within 500 ft. (Influence includes changing heading or speed.)

The concept developer is to look at three instantiations of the SoS, one using no communications or sensing of other UAS (SoS1), one using sensing only and no communications (SoS2), and one using full communications and sensing (SoS3). Environmental information I_e is constant for all SoS, as is the scenario distribution. For any given pop-up threat, its probability of occurrence within the overall scenario distribution is constant across the different SoS.

4.6.3.1 SoS1.

SoS1 does not use the cooperative algorithm under consideration, since there is no means to make the UAS cooperate. Coverage is accomplished using synchronization, where each UAS flies a prescribed path optimized to provide full coverage. This technique works for nominal scenarios, but for unplanned obstacles or pop-up threats, such an algorithm is unable to recover.

As compared to an adaptive system, there will be a greater number of pop-up threats for which the SoS behavior response will be unable to counter. This implies a greater number of events that make up the set Y_a, such as in equation 25.

Since $I_{ij} = 0$ for all UAS, per equation 39 $\mathcal{F}_X(ai, L, \tau, \Psi_{ai}) = \sum_{\substack{j=1 \\ j \neq i}}^{n} \mathcal{F}_{W_j^c}(ai, L, \tau, \Psi_{ai})$.

4.6.3.2 SoS2.

SoS2 does use the cooperative algorithm, but as the UAS are only configured with sensors and no communications, there is no potential for full knowledge of the activities of every other UAS. For this SoS, I_{ij} will be in the range of 0.2 to 0.5 (representing a mid-range knowledge); depending on the conditions relative to the two aircraft for each I_{ij} calculation, the knowledge content will vary.

$\mathcal{F}_X(ai, L, \tau, \Psi_{ai})$ for SoS2 will be higher than that of SoS1, owing simply to the fact that the I_{ij} term is non-zero.

Also, with the ability to adjust SoS behavior in response to changes among the constituent UAS, the set Y_a will be smaller, as there are more situations from which SoS2 can recover.

4.6.3.3 SoS3.

SoS3, equipped with full communications and sensors, will have I_{ij} will be in the range of 0.8 to 1.0 in this example. $\mathcal{F}_X(ai, L, \tau, \Psi_{ai})$ for SoS3 will be higher than either

that of SoS1 or SoS2. Similarly, set Y_a will be smaller than both other instantiations.

4.6.3.4 Comparison of Success Probabilities.

Given that $\mathcal{F}_{X3} > \mathcal{F}_{X2} > \mathcal{F}_{X1}$ and $\mathcal{F}_{Y3} < \mathcal{F}_{Y2} < \mathcal{F}_{Y1}$, per equation 43 it can be determined that the lower bound of $P_{success}(SoS3)$ is less than the lower bound of $P_{success}(SoS2)$ is less than the lower bound of $P_{success}(SoS1)$. These conclusions directly inform the designer of the effectiveness of design choices in relatively quick order.

In a different perspective of this problem, consider what occurs under degradation of the communications network of SoS3, such as due to adversary action. A steady reduction in the functionality and probability of success for the SoS is apparent first with loss of the communications network (e.g. due to jamming) and then with loss of inter-flight sensing capability (e.g. due to obscurants or directed energy).

4.6.4 Information Overload.

Information overload, as discussed earlier, is defined here as a form of decision paralysis characterized by the inability to respond to a disturbance due to insufficient processing of relevant information or overprocessing of irrelevant information. As discussed, there is a cost to collecting and processing information.

If Z_a is the event that the information collected provides no increase in the probability of mission success, $P(Z_a)$ is the probability that this event has occurred. Thus the term in equation 46 captures the effect of collecting useless information. It is assumed the probability of event Z_a is independent of the scenario distribution. In contrast to $\mathcal{F}_X$, $\mathcal{F}_Z$ is the expected functionality conditioned on the events where collected information provides no increase in mission success.

$$\mathcal{F}_Z(a, L, \tau, \Psi_a) \equiv \sum_{\Psi_a} \mathcal{F}(a, L, \tau, S_i|Z_a)\ P(Z_a)\ p(S_i) \tag{46}$$

The goal of the system is to collect as much information on the environment as possible that is relevant to the success of the mission, rejecting all other information. The environment, if it has a goal (i.e. a competitor to the first system), seeks to either withold useful information or provide useless information. Using this framework, it may be possible to quantify and predict information overload and evaluate the effectiveness of system designs.

4.6.5 Implications for system design and effectiveness analysis.

The earlier sections demonstrate that bounds on the probability of success or probability of survival (survivability) for a system can be calculated using information on the expected functionality of the system and the environment. In fact, this can be found using *only* the functionality estimations, without directly simulating the system within the specific environment under consideration.

This has important practical implications. Effectiveness analysis is often a costly and time-consuming process required at multiple phases within a development program. The ability to implement a "quick turn effectiveness" tool which produces Rough Order of Magnitude (ROM) calculations for the effectiveness of large numbers of system concepts within a particular environment would serve to significantly reduce time and resource requirements. Using the approach in this paper, prior simulations of a system in a different environment can be used to calculate the survivability and probability of success within the environment of concern, without direct simulation. Provided that the functional level L and epoch size τ are consistent, functionality estimates for the system and functionality estimates for the environment can provide that ROM estimate.

These calculations may also be performed for a system of interest against an adversary system, again without direct simulation. Suppose a designer is interested in the performance of a UAS against a new Surface-to-Air Missile (SAM) threat. Using estimates of the UAS's expected functionality against an older SAM system and estimates of the new SAM system's expected functionality, the estimated survivability of the UAS against the new system can be calculated directly.

The effects of independent systems being used cooperatively may also be investigated using this functionality framework. Using estimates of the influence of one cooperative system on another, the survivability and probability of success for the combined system can be calculated.

4.7 Application Simulation: Competitive and Cooperative UAS

In order to demonstrate potential application of the theory developed in this research, a simple simulation was developed. The simulation represents a design problem for a UAS with a mission of traveling to a target waypoint while avoiding detection. In the defined framework for this example, the UAS starts at point (0,0), can only travel in the 'north' or 'east' directions (no back-tracking), can only move in one direction at a time (no diagonals), and ends at point (3,3). The mission is failed if the UAS passes through point (2,2), the location of the simulated detector.

Figure 10 presents an example successful mission where the UAS traveled 'north' from (0,0) to (0,1), then 'east' to (1,1), and eventually to the target at (3,3). The UAS may only travel north or east one unit per time step, allowing for exactly 6 moves per simulation.

As structured, the simulation allows for a maximum of $\binom{\text{total moves}}{\text{moves north}} = \binom{6}{3} = 20$ possible paths. Of those, 8 result in mission success as defined, and 12 result in failure. Figure 11 illustrates an overplot of all possible successful paths and Figure

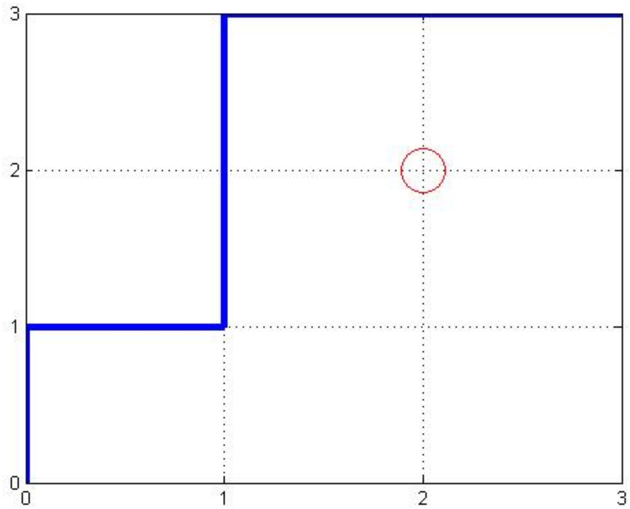

Figure 10. An example path resulting in mission success. The UAS avoids the detector at (2,2) before reaching the target point at (3,3).

12 does the same for paths resulting in failure.

The scenarios are not distributed uniformly; due to conditional probabilities of transition at each point it turns out that (overall) it is more likely for the UAS to follow a success path than a failure path. For an unconstrained UAS, the expected functionality is $F_X = 7.125$ per equation 14 and has a probability of success of $P_{success} = 0.625$ (these are calculated exactly). (Note that if the scenarios followed a uniform distribution, $P_{success} = \frac{8}{20} = 0.4$.) The maximum possible functionality is 12 and the minimum is 4.

4.7.1 Cooperative UAS.

4.7.1.1 Energy-constrained UAS.

Now assume the UAS is provided with enough fuel for only 4 functional interactions. That is, each time the UAS performs a maneuver (a stop, start, or turn), it expends one unit of energy. If the chosen path requires more than 4 units of en-

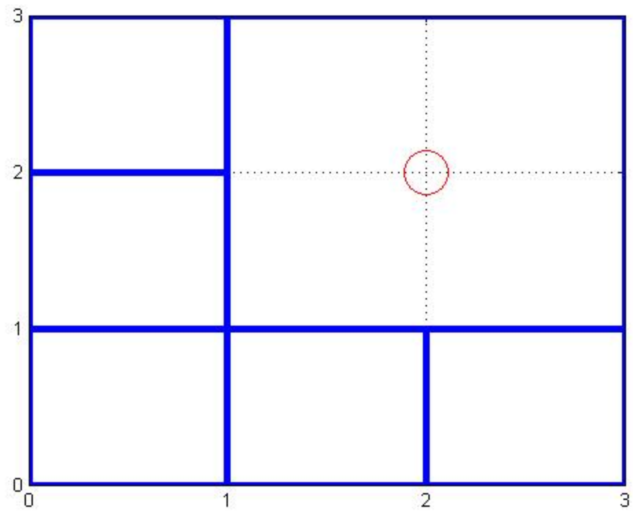

Figure 11. All possible scenarios resulting in mission success.

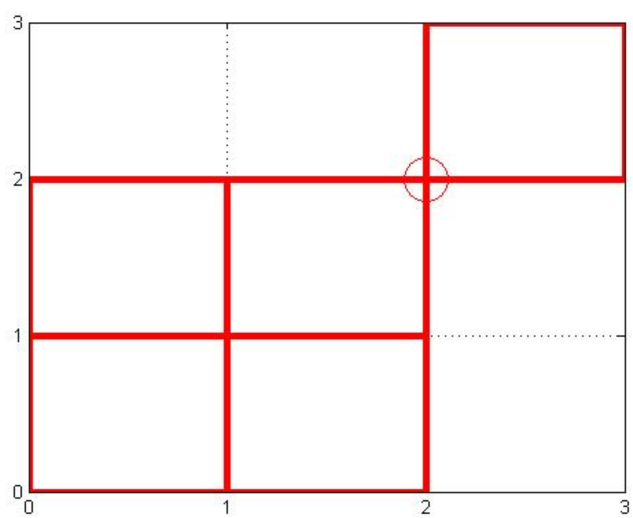

Figure 12. All possible scenarios resulting in mission failure.

ergy to reach the target, the UAS runs out of fuel and the mission is failed. Under this constraint, the expected functionality for the UAS is $F_X = 4.000 \pm 0.001$ (at 95% confidence) since the UAS can have no more than 4 functional interactions. $P_{Success} = 0.26$, since there are only two paths where the end point is reached successfully. (Note that uncertainty is not given for $P_{Success}$ since it is calculated as true or false for each scenario, not as a statistical quantity.)

Now suppose a second UAS can influence the first through refueling; the first UAS gains functionality due to the increased energy available through the added fuel. In this case, every time the two UAS are in proximity to each other, UAS 2 transfers 1 unit of energy to UAS 1. The is expressed as an increase in potential functionality of 1 when both UAS occupy the same point on the grid. UAS 2 also starts from (0,0) and moves along one of the 20 paths to (3,3).

If UAS 1 has no information on UAS 2 (and vice versa), then I_{12} in equation 38 is zero and only the second term applies. Taking into account now all the possible path permutations for both UAS, this yields an expectation of an extra 2.0 units of energy (meaning an average possible increase in functionality of 2.0). The simulation produced an expected functionality for UAS 1 of $F_{W_2^c} + F_{W_1^c} = F_X(UAS1) = 5.611 \pm 0.016$ (with 95% confidence). The probability of success increases to $P_{Success} = 0.38$. This is in agreement with equation 43, which states that the floor value (minimum probability of success) should increase.

4.7.1.2 Cooperative UAS with Increased Information .

If UAS 1 has information on the position of UAS 2, through a combination of communcation and observation, I_{12} is positive and F_X for UAS 1 increases. Assume that the state of UAS 1 includes only a position. (An expanded treatment would include the velocity vector as well, but that is not necessary for the purposes of this

example application.) The quantity I_{12} can be stated in terms of entropy: $I_{12} = 1 - \frac{H_{12}}{H_\infty}$; in other words, I_{12} is 1 minus the uncertainty in the position of UAS 2 divided by the total number of possible states for UAS 2 (in this example, there are 16 possible points UAS 2 could occupy). If UAS 1 has perfect knowledge of UAS 2's position, $I_{12} = 1$, and every path will have the maximum possible increase in available energy (5 units; 1 unit every time step). Assuming UAS 1 has power to act on that information, this increases the likelihood that UAS 1 will choose paths (scenarios) that maximize the time spent in proximity to UAS 2.

Referring to equation 38, under perfect knowledge $F_X(UAS1) = 6.816 \pm 0.018$ (with 95% confidence) after simulation. The probability of success is the same as the full energy case, 0.625. The paths which did not gain enough functionality to reach the target (those with $F > 9$) failed originally anyway because all of those paths pass over the detector at (2,2).

4.7.2 Competitive UAS.

4.7.2.1 Information-constrained UAS.

Now suppose that the mission of UAS 2 is to intercept UAS 1, which causes an additional failure mode for UAS 1. In this situation, UAS 2 starts at (3,3) and its movement is subject to the similar constraints as UAS 1 except that UAS 2 can travel in all four directions (including the 'south' and 'west' directions). (Note that UAS 2 will have higher functionality with the removal of these constraints.)

If UAS 1 and UAS 2 occupy the same point on the grid, UAS 2 is successful and UAS 1 fails. Similarly, if UAS 1 passes the detector at (2,2), UAS 2 wins and UAS 1 fails. Assume first that UAS 2 has no information on UAS 1 and therefore moves randomly. After running a number of simulations, $P_{success}(UAS1) = 0.52$ and $P_{success}(UAS2) = 0.48$. This is lower than the 0.625 for $P_{success}(UAS1)$ from earlier,

as expected with an additional failure mode.

4.7.2.2 Competitive UAS with Increased Information.

UAS 2 now is given information on the current position of UAS 1, with no uncertainty. In this case $I_{21} = 1$ and UAS 2 intercepts UAS 1 every time due to its higher expected functionality.

If instead of perfect information, UAS 2 knows the position of UAS 1 only within 1 point (out of 16 possible points on the grid). If UAS 1 is in a corner point, there are 3 possible points for it from the perspective of UAS 2; if UAS 1 is in the center of the grid, there are 5 possible points. $I_{21} = 1 - \frac{H_{21}}{H_\infty} > 1 - \frac{5}{16} = 0.6875$. This results in $P_{success}(UAS1) = 0.34$ and $P_{success}(UAS2) = 0.66$, giving UAS 1 a fighting chance.

4.8 Conclusions and future work

As engineered systems, particularly military acquisitions, increasingly rely on information about their environment, devising strategies for increasing system resiliency is of critical importance. This paper provides a foundation for understanding the effects of information on system functionality using a metric that can be employed practically during system design.

The derivations explored in this paper can help guide development of new systems in order to increase their chances of success in the target environment. Significant further work is required, however, to construct practical design guidance.

Extensions of this work have important implications for information overload, competitive systems design, and cooperative systems design. Future efforts will include exploring linkages between information, functionality, complexity, decline, and collapse, all of which are of high interest for systems design.

V. Application

In order to explore and validate the theories and assertions developed in Chapters III and IV, an expanded version of the simulation used in Chapter IV was developed. The purpose of the simulation is to meet the following objectives:

- Show that higher functionality corresponds with higher probability of success;
- Show the effect of limited available energy on functionality and probability of success;
- Show the effect of limited information on functionality and probability of success.

5.1 UAS Simulation

As in Chapter IV, the simulation represents a UAS traveling to a target waypoint while avoiding detection. In the expanded version, the UAS starts at point (0,0), can only travel in the 'north' or 'east' directions as before, can only move in one direction at a time, and ends at point (20,20). The mission is failed if the UAS passes through a detector. The number of detectors is varied from 0 to 10. Figure 13 illustrates the set of success paths for an example obstacle distribution.

For 21 x 21 grid, the simulation allows for a maximum of $\binom{\text{total moves}}{\text{moves north}} = \binom{38}{19} = 3.5345x10^{10}$ possible paths. The maximum functionality for any path is 74, and the minimum is 4 as before.

5.2 Results

A simulation of a single UAS (without a refueling companion) was run to examine the effects of constraining the available energy. The simulation was configured for

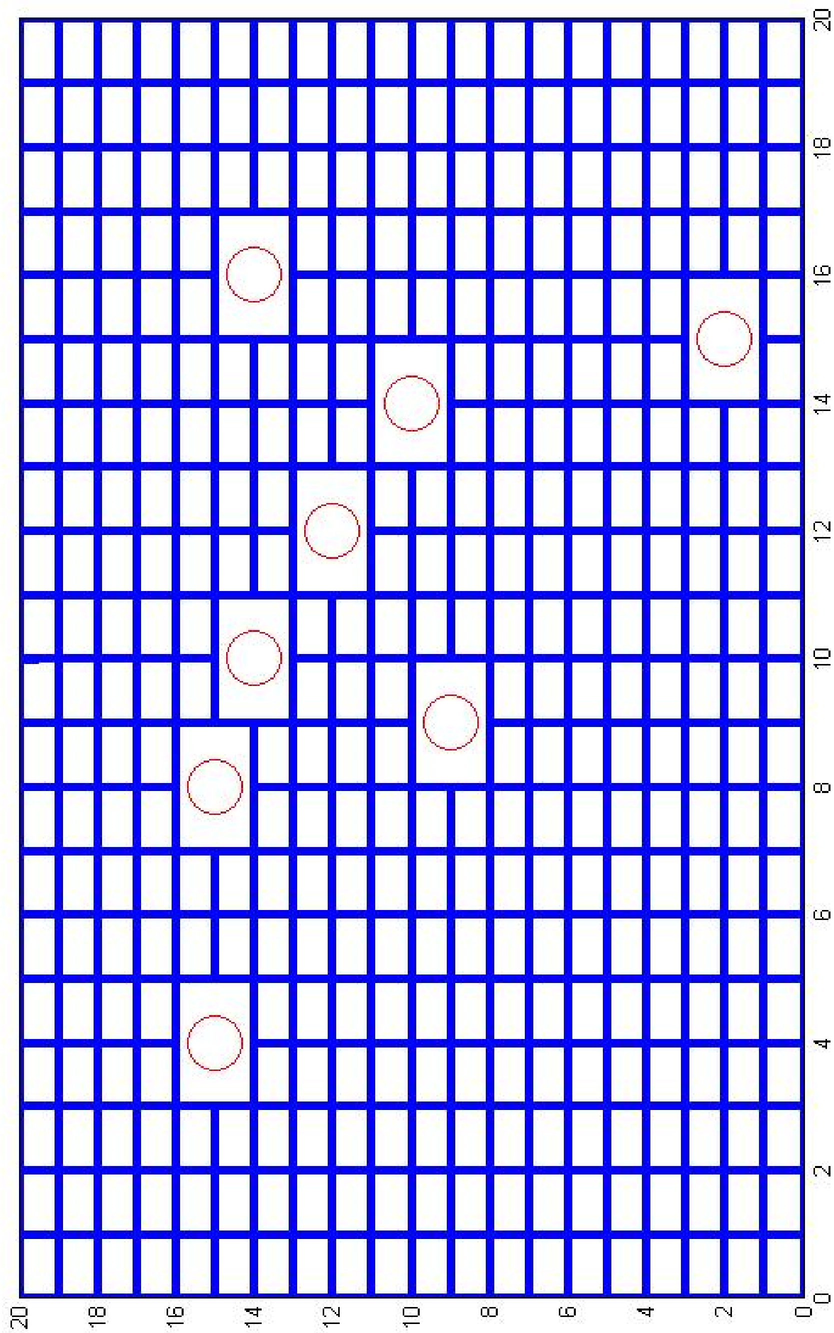

Figure 13. Scenarios leading to mission success

20,000 runs and no obstacles. Figure 14 shows the results for various initial energy conditions (the 95% confidence error bars are barely visible).

There is a clear trend in that the expected functionality decreases smoothly with decreasing energy. This is in agreement with the assertion of Chapter IV, which stated: *If all other parameters are held constant, as the energy available to a system increases, the system will maintain or increase in expected functionality, provided that the destructive energy limit for the system is not exceeded. The converse is also true: as the energy available to a system decreases, the system will maintain or decrease in expected functionality.*

The trend between probability of success and expected functionality is also clear. At high functionality, there is a sharp drop in success probability with small changes in functionality. At low functionality, the opposite is true: large changes in functionality yield small changes in probability of success.

The simulation was run with ten obstacles placed according to Table 3. Figure 15 shows the results for various initial energy conditions (the 95% confidence error bars are barely visible), with a refueling tanker delivering an energy boost of 1 unit. The UAS and tanker follow random (zero information) paths.

Table 3. Obstacle positions

Obstacle Number	Coordinates
1	(9, 9)
2	(12, 12)
3	(14, 10)
4	(10, 14)
5	(8, 15)
6	(16, 14)
7	(4, 15)
8	(15, 2)
9	(12, 6)
10	(6, 12)

As with the zero obstacle case, the expected relationships between available energy,

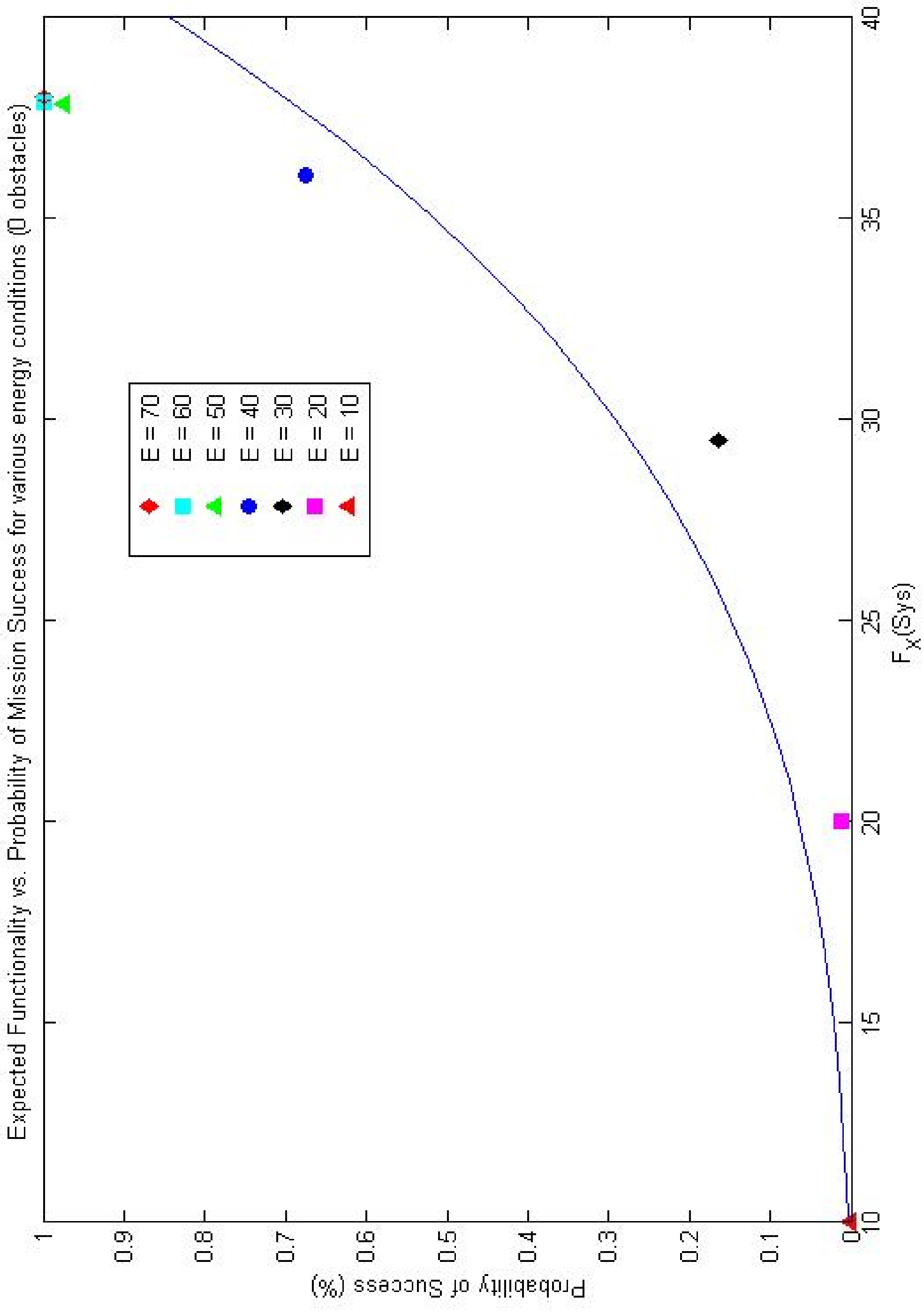

Figure 14. Expected Functionality vs. Mission Success for a variety of energy conditions and no obstacles

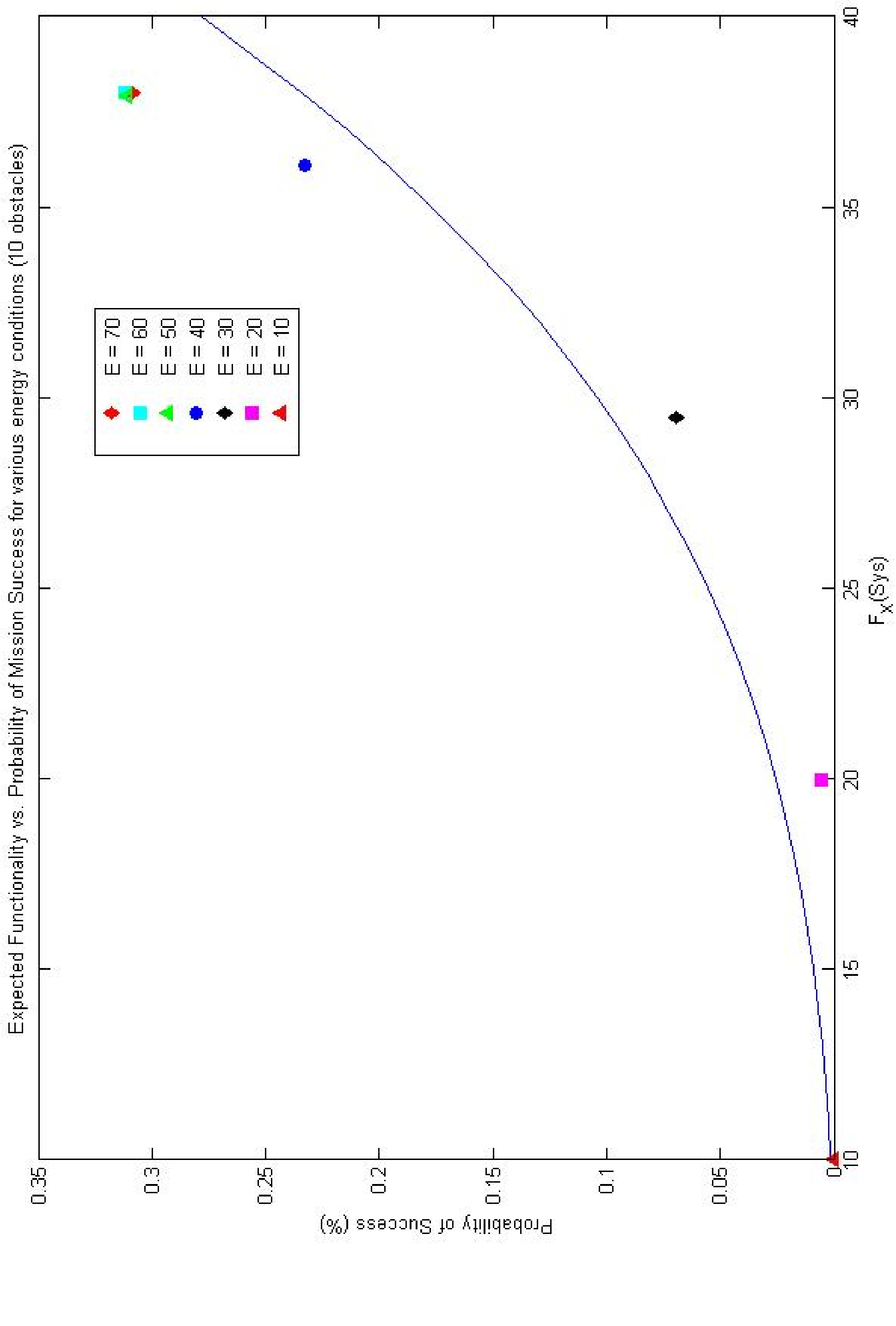

Figure 15. Expected Functionality vs. Mission Success for a variety of energy conditions and ten obstacles

expected functionality, and probability of success are clearly evident. The effects are less pronounced, however, in the ten obstacle case. At high functionality, the probability of success is much less sensitive to changes in functionality for the ten obstacle case than the zero obstacle case. At low functionality, the 10 and 0 obstacle cases approach the same sensitivity to functionality changes. For clarity, the scenarios with 0 obstacles and 10 obstacles are overlaid in Figure 16.

It is clear that there is an overall flattening of the expected functionality vs. probability of success as the number of obstacles increases. This is due primarily to the reduction in the maximum possible probability of success with the increase in obstacles; with a lower maximum, there is less range for the effect of reducing expected functionality to be seen.

Next, the refueler was simulated for different values of energy delivered upon each encounter with the first UAS. Figure 17 shows the effects of varying the amount of energy delivered by the refueler.

As expected, a boost energy of 3 results in the highest overall higher expected functionality and probability of success. UAS 1 needs relatively few encounters in order to have enough energy to reach the target point. This result further reinforces the assertion that increased energy available yields increased expected functionality.

In order to explore the relationsihp between functionality and information, an uncertainty in the position of the refueler (as observed by UAS 1) was introduced. For an initial available energy of 20, Figure 18 shows the results for various uncertainties in the information UAS 1 has on the refueler.

As in the 4 x 4 grid example, this result is consistent with equation 43, which states that the floor value (minimum probability of success) should increase as I_{12} increases. As the uncertainty approaches infinity (which was calulated by setting I_{12} to zero), probability of success appears to become slightly less sensitive to decreases

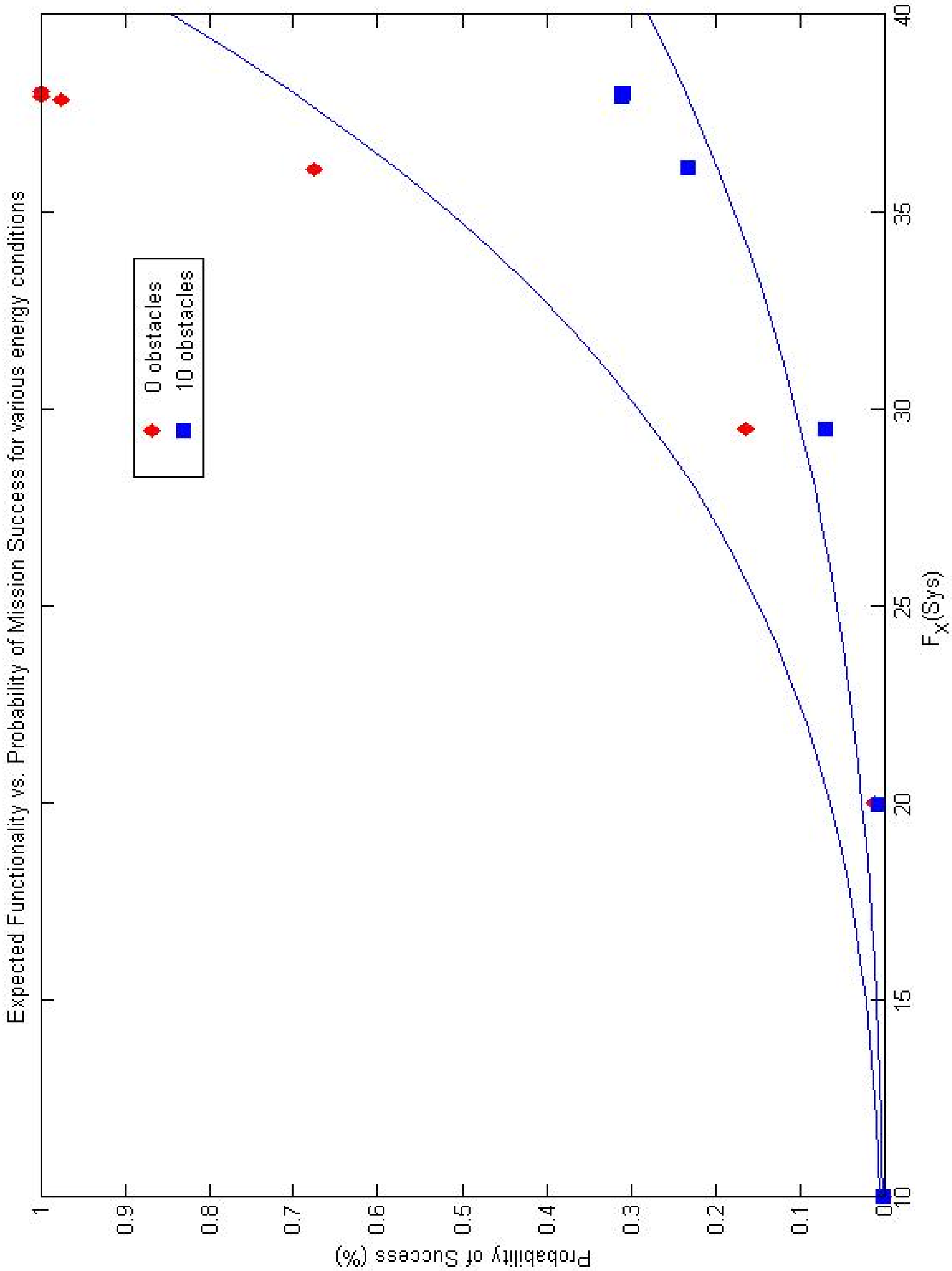

Figure 16. Expected Functionality vs. Mission Success for a variety of energy conditions, for 0 and 10 obstacles

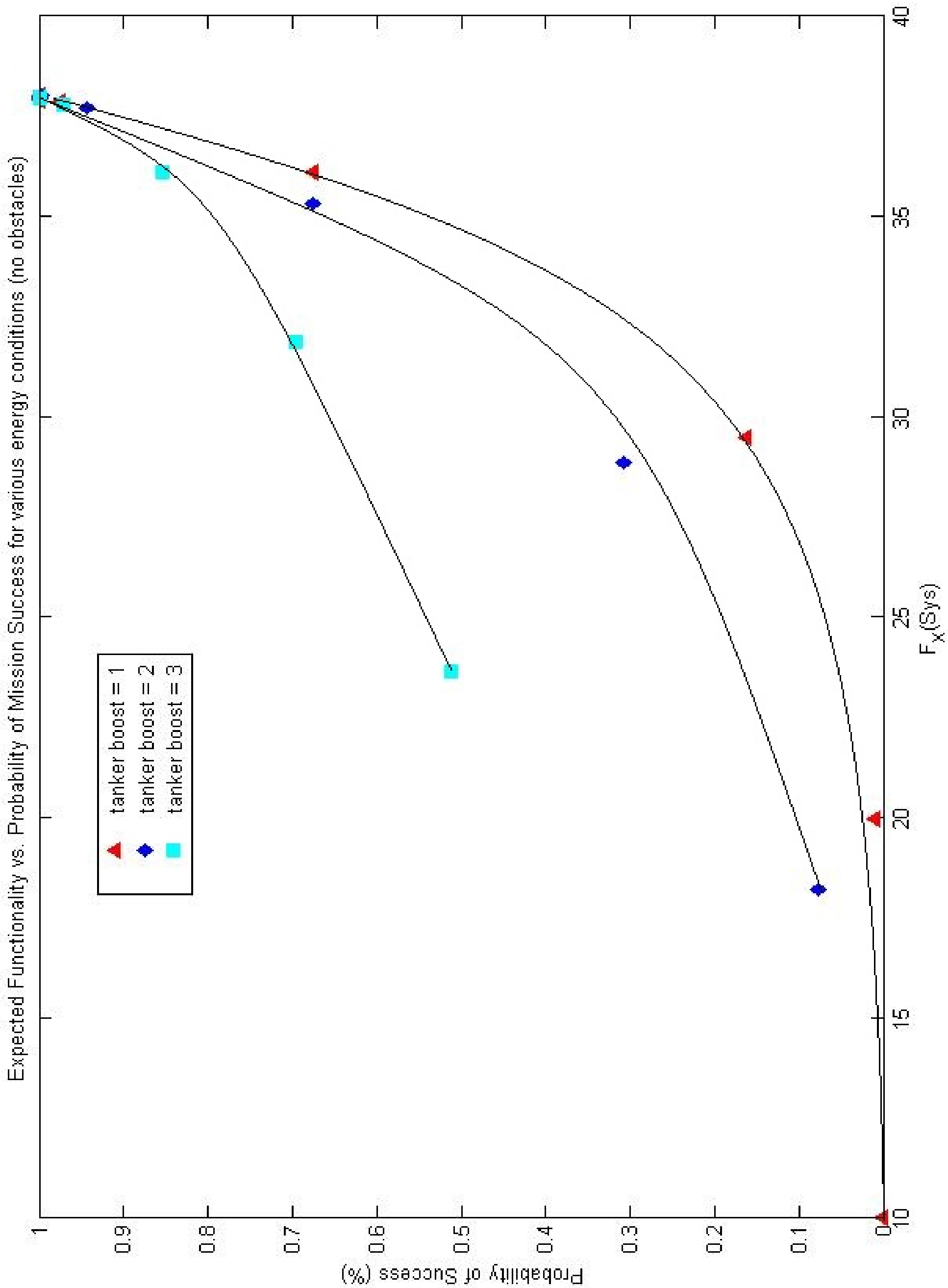

Figure 17. Expected Functionality vs. Mission Success for multiple tanker boost energies

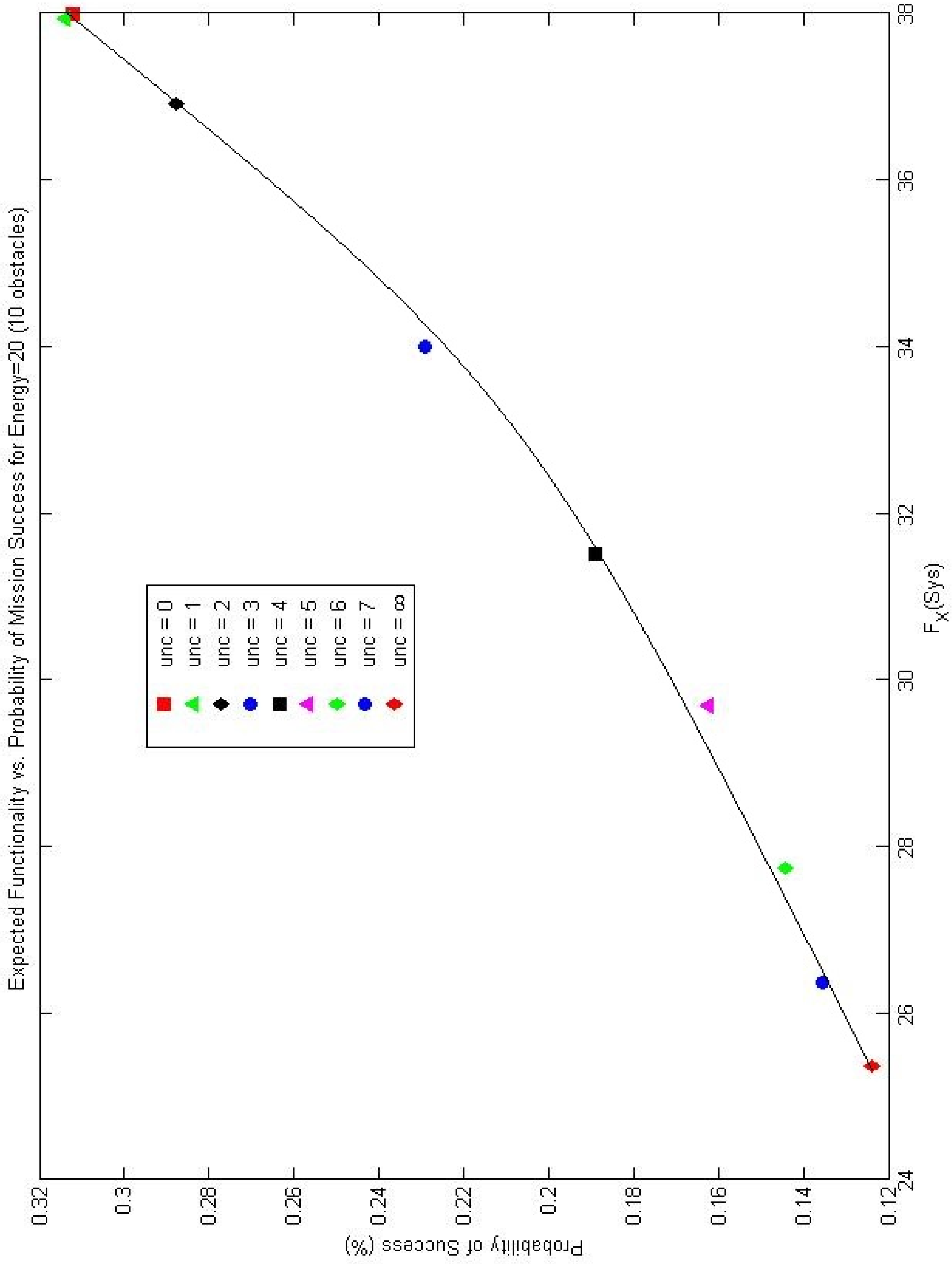

Figure 18. Expected Functionality vs. Mission Success for a variety of uncertainty conditions and ten obstacles

in expected functionality. The trend, though, appears nearly linear.

Interestingly, due to the construct of the simulation, as uncertainty continues to increase beyond the theoretical limits the probability of success increases. This is illustrated in Figure 19 Since the uncertainty in position of the tanker increasingly includes space outside of the allowable grid, the UAS is pulled toward the edges of the grid. Because those low-functionality paths are less populated with detectors (obstacles), they are more likely to be successful. This illustrates how the idiosyncracies of the particular problem under consideration can lead to counter-intuitive behavior.

5.3 Summary

This chapter successfully demonstrated that the expected trends hold for the simulation used. Expected functionality clearly decreases (or stays the same) in response to changes in available energy. Probability of success clearly decreases as expected functionality decreases. Both functionality and probabilty of success fall as the information available to a system is reduced.

While the theory holds for this system construct, future work should include conducting further validation with a variety of system types.

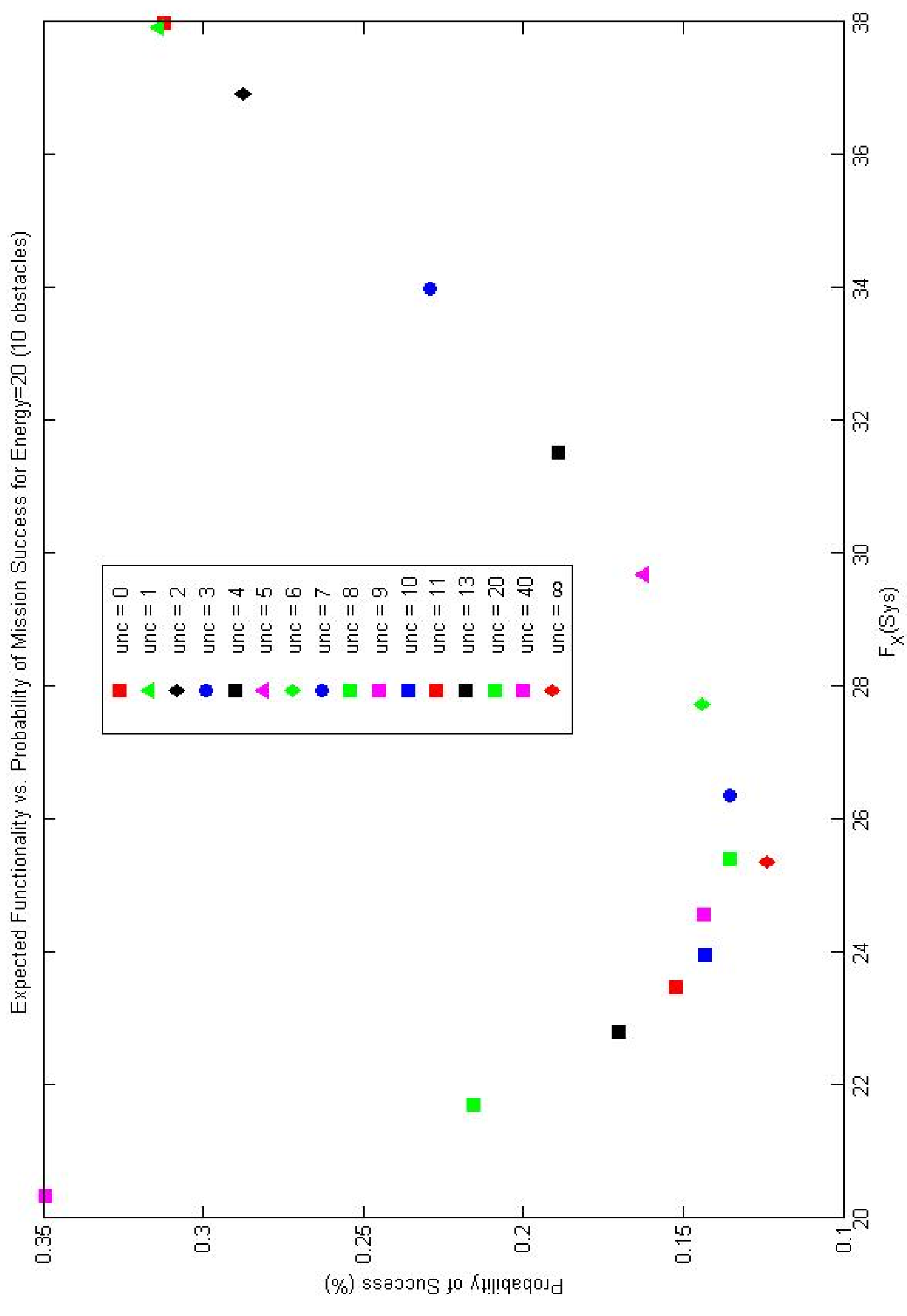

Figure 19. Expected Functionality vs. Mission Success for a variety of uncertainty conditions and ten obstacles

VI. Conclusions and Recommendations

6.1 Conclusions

This research successfully developed a methodology to measure the functionality and complexity of dynamic systems for use inevaluating resilience or robustness of a proposed concept. The research extended and integrated existing theory and measurement methodologies into a practical approach to aid rapid effectiveness analysis for resiliency.

The work presented here successfully addressed the research objectives discussed in Chapter I, which were to answer the following questions:

- How can functionality be measured?

  – This research demonstrated that functionality is measured by determining the number of functional interactions, an aspect of the behavior of the kinetic energy of a system element in each degree of freedom. The expected functionality for a system can be calculated using the realized functionality for multiple scenarios in context with the probability distribution for the scenario set.

- How can complexity be measured and what is its relationship to functionality?

  – This research identified various types of complexity from the literature and demonstrated that expected functionality, which measures the behavioral complexity of a system, may be used as a proxy for measuring complexity. The measure aligns with the accepted approach of measuring complexity by calculating the information content of a system.

- What is the effect of reduced energy availability on functionality and complexity?

– This research developed validated theory that a decrease in energy availability leads to a decrease in functionality, and therefore complexity.

- What is the effect of reduced information on functionality and complexity?

  – This research developed validated theory that as a system's information on its environment (or a competitive system) increases, the probability that a system will apply the most suitable functions increases, provided that the cost of obtaining that information is much less than the energy available. This leads to a higher functional efficiency, increases the probability of success. Lower information increasing the likelihood of poor application of functions and encountering destructive effects, lowering functionality (and complexity).

- How can knowledge of functionality and complexity be applied to the systems engineering process?

  – This research demonstrated the utility of using measurement of functionality and complexity to guide systems design toward resiliency. The expected functionality metric can be used to select systems that are more likely to succeed across a diverse range of scenarios. Expected functionality may be used as a "quick turn effectiveness" methodology to rapidly identify the most suitable system approaches.

While this research successfully met the research objectives, there are areas where the conclusions could be stronger. A greater variety of applications with more inherent variety of behavior would allow for more conclusive validation.

6.2 Contributions

This research resulted in several contributions. First, it established a theoretical basis for the measurement of functionality and its use in system design. This

measurement was linked with the notion of complexity and offered approaches for its management. This research developed validated derivations of the impacts of energy and information scarcity on system behavior and effectiveness. The functionality metric was used to derive theoretical impacts on the probability of mission success, taking into account available information, cooperative behaviors, and competitive behaviors. Finally, this research outlined and demonstrated a methodolgy for applying the theoretical approaches to aid in system design.

6.3 Recommendations for Future Research

Continuing work should focus on further validation of the theory with a variety of system types and an increased number of system elements, as well as including models with more degrees of freedom. Future research should also validate the expected effects of information overload on functionality and establish approaches to counter it.

More conclusively linking functionality to complexity would strengthen the theoretical foundation for this work. This research effectively argued the parity between the concepts and identified correlation using one accepted method for measuring complexity. Examination and comparison using other complexity measurements would allow further linkage between functionality and the rich field of complexity.

Chapter II explored a number of existing approaches to anticipate system collapse. In particular, work by Scheffer, et. al [57] identified that slowing in the rate of change in state variables, an increase in autocorrelation coefficient, and increase in variance. Recalling Tainter, [65], collapse is defined as a rapid reduction in complexity. Further research should investigate using changes in functionality (complexity) itself over time as an indicator for impending collapse, including changes in eignevalues, autocorrelation coefficient, and variance of functionality. No known prior research

used measures of the complexity or functionality of the system in order to predict collapse. More accurate anticipation of collapse could allow for the application of preventive measures or mitigations, such as developing alternatives to the USAF acquisition system [47].

Finally, large-scale application of expected functionality to a large number of systems of different types would enable the development of design heuristics with respect to functionality. This research established fundamental guidance with respect to functionality, energy, and information, but there are likely many more unexplored design principles building upon this work.

Appendix A. Early work: Practical measurement of complexity in dynamic systems

This appendix is a paper developed in the course of this research which was reviewed, accepted, and published in the proceedings of the 2012 Conference on Systems Engineering Research [17]. This paper represents an early point in the evolution of this research prior to incorporating ideas on functionality and stronger ties to systems engineering methods (rather than solely focused on thermodynamic complexity).

1.1 Abstract

A difficulty in complexity theory is lack of a clear definition for complexity, particularly one that is measurable. Those approaches that provide measurable definitions for the absolute complexity of a system often impose the requirement of perfect or near-perfect knowledge of system structure.

In practice, it is intractable or impossible to measure the complexity of most dynamic systems. However, by measuring behavioral complexity in context with environmental scenarios, it is possible to set bounds on a system's absolute (maximum) complexity and estimate its total complexity. As this paper shows, behavioral complexity can be determined by observing a system's changes in kinetic energy.

This research establishes a methodology for measuring complexity in dynamic systems without the requirement of system structure knowledge. This measurement can be used to compare systems, understand system risks, determine failure dynamics, and guide system architecture.

1.2 Introduction

A difficulty in complexity theory is a clear definition for complexity, particularly one that is measurable. In many cases throughout the literature, it seems the authors are speaking of different or even mutually exclusive phenomena. The most apt summary of the issue is the tongue-in-cheek assertion by complexity researcher Seth Lloyd: "I can't define it for you, but I know it when I see it." [20]

1.2.1 Definitions of Complexity.

A root cause in the lack of unified complexity definitions is that there are in fact several types of complexity. The first formal treatment of complexity focused on algorithmic complexity, which reflects the computation requirements for a mathematical process. [20] Senge and Sterman include also dynamic complexity, which is primarily characterized by difficult to discern cause-effect relations. [59] [62] To further muddy the waters, there is the concept of complex adaptive systems. These are defined by properties such as consisting of agent networks, having multiple levels of organization, development of internal models, exhibiting 'phase transitions', and exploitation of niches in the fitness landscape. [67]

One of the most workable definitions is thermodynamic depth, which actually seems to unify the two complexity camps. Thermodynamic depth as complexity asserts that complexity is a measure of how hard it is to put something together. [**?**] [**?**] There are several variations on this approach, with the commonality that complexity disappears for both ordered and purely random systems. [44] [20]

Bar-Yam defines the complexity of a physical system as the length of the shortest string, s, that can represent its properties. This can be the result of measurements and observations over time. [5]

However, there are some inherent difficulties in using these definitions. For one, it

is very difficult to approach a macro-level system, particularly one with hidden structure, and determine the information content. While information and thermodynamic entropy-based measures form a theoretical basis for complexity measurement, there remains a need for a more accessible route.

There are hints that energy plays a role in defining complexity. As a general rule, systems commonly recognized as complex process more energy than less complex ones. For instance, civilizations move more energy than a single town. However, mere energy flow is not a measure of complexity, since a single arbitrary large-bore "pipe" (a relatively simple system) could be constructed that matches the energy flow through any complex system.

A metric that solves the large-bore problem is proposed by Chaisson. By measuring the energy rate density in Equation 47, where Φ_m is the energy rate density, E is energy flow through a system, τ is the time epoch, and m is system mass, Chaisson obtains results that correlate well with other notions of complexity. [13]

$$\Phi_m = \frac{E}{\tau m} \tag{47}$$

However, the energy rate density metric has some drawbacks. By normalizing with respect to mass, this metric produces incorrect results for complexity when comparing some systems. For example, suppose an electronic brain is built to mimic the operation of a human brain. The human brain may process energy at the same rate as a theoretical electronic brain, but due to differences in basic materials (i.e. the weight of neurons vs. semi-conductors), the two systems, which most theorists would recognize as identically complex, could have vastly different Φ_m values. Thus, by normalizing with respect to mass instead of function, the Φ_m metric produces incorrect results for the relative complexity of systems.

A practical difficulty in using the Φ_m metric is determining the appropriate mass

and energy to use. In measuring the Φ_m of a civilization, Chaisson uses the mass of humanity and the total energy processed by the civilization. However, the total energy of a civilization does not flow through only its humans, but also its machinery, beasts of burden, vehicles, etc., the mass of which is a difficult quantity to measure.

All metrics discussed thus far contribute ideas toward a measure that:

- Correlates with notions of complexity at all scales
- Is measurable for any dynamical system
- May be practically employed

1.3 Energetic Complexity

Dynamic systems, by definition, yield changes in their kinetic energy over time. Using this fact, we can define an Activated Energy Transfer as an instance where, for a discrete element of a system, its kinetic energy in the direction of a generalized coordinate transitions from a non-zero value to a zero value, or from a zero to a non-zero value.

As an example, given a particle enclosed in a container, every collision with the sides of the container is considered an activated energy transfer. The definition of activated energy transfers allows the following definition of Energetic Complexity: the number of activated energy transfers for a given system within a particular epoch and above a particular functional level. It is denoted by Ç.

This is, in essence, complexity by demonstration. A static system may have potential for complex behavior, but its complexity cannot be measured until it is active. Depending on the scenario, a system may respond with differing amounts of complexity. A plane flying through clear skies may stay straight and level, but be required to make complex maneuvers when flying through a storm.

A key to the utility of this definition is the ease with which it may be applied. For instance, if we choose an epoch size larger than the time required for basic cell functions, a humans internal structure is dependent on exercising a finite number of energy transfers. That is, it must ensure a constant flow of energy within the body, and perform work using that energy, in order to remain alive. If energetic complexity is reduced, it means that part of the body is processing less energy – the tissue is either reduced in its function (e.g. a sleeping brain), is dead, or is removed.

Important to this definition is functional level. Specifying the functional level (the threshold above which a system function can be realized) relates the measurement to a particular level in the hierarchy of system functions. For instance, a humans complexity may be measured at the cellular, molecular, or atomic levels, and given the same epoch size, the Ç. value increases for each progressively smaller scale. The functional level of interest might be different between generalized coordinates and is analogous to energy levels.

It should be noted that other researchers have alluded to both the multi-scale and functional natures of complexity, but there is no apparent prior synthesis of these ideas. Bar-Yam has written extensively on multi-scale complexity and what he terms the "complexity profile", applying it in particular to warfare. As he writes, "complexity at a particular scale includes all possible force actions at or above this scale." [7]] In his book on complexity, systems engineer Suh argues that complexity can only be defined in the functional domain vs. the physical domain, and relates complexity to the ability to satisfy a functional requirement. [64]

1.4 Consistency with the Field and Further Extension

While the concept of energetic complexity represents a new approach in looking at complexity, in order for it to be shown valid, there must be coherence and consis-

tency with the accepted definitions of complexity in literature. Note that energetic complexity measures the dynamic complexity of a system.

As explored earlier, complexity is often defined as a deep property of a system that is in many ways intangible. However, as pointed out by Corning, such definitions "exclude the extremes associated with highly ordered or strictly random phenomena, even though there can be more or less complex patterns or order and more or less complex patterns of disorder." [20] In short, these definitions do not allow for graduations in complexity. In rejecting "simpler" systems as not being complex, these definitions eliminate the very basis for which to determine whether a system is complex or not!

"Deep-property" definitions of complexity reflect a combination of energetic complexity and system structure. For example, a linear dynamic system (e.g. where cause and effect are apparent) can have the same Ç value as a network (Figure 20). However, complexity theorists are more likely to describe a network as complex than a linear system. The main difference between these systems is the path in the linear system is predictable, whereas the network path is dependent on additional factors whose properties must be known to make predictions. It is the difference between the absolute complexity or the total complexity of the two systems.

Figure 20. Two systems with identical realized functionality. (a) Single-path (b) Multi-path Network

Absolute complexity is defined here as the maximum possible complexity for any given scenario over the range of all possible scenarios, S_m, as in Equation 48. Energetic complexity may be used to measure absolute complexity. If it was possible to observe a systems response to all possible scenarios and measure the energetic complexity,

this information could be used to construct a description of the absolute complexity of the system. As the certainty of a system's structure and function decrease the certainty in (and ability to measure) absolutely complexity is also reduced.

$$Ç_A = max(Ç|S_m), S_m \in \Psi \tag{48}$$

Energetic complexity, as defined above, is a single observation of system responding to a particular scenario.**Total complexity** may thus be written as Equation 49 where a particular scenario S_m is defined as a unique set of system conditions (exogenous and internal), and Ψ is defined as the set of all possible scenarios driving a unique system response. F is the Cumulative Density Function (CDF) of Ψ.

$$Ç_A^{sys} \equiv E[Ç(sys, \Psi)] = \int_\Psi Ç(sys, S_m)dF(S_m) \tag{49}$$

For a set of finite, discrete, and independent scenarios, total complexity may be written as in Equation 50 where $p(S_m)$ is the probability mass function over Ψ.

$$Ç_T^{sys} = \sum_\Psi Ç(sys, S_m)p(S_m) \tag{50}$$

For a given system, one scenario may cause it to respond with a particular behavior, whereas another may produce a completely different behavior. As defined here, scenarios which produce identical behavior in the system(s) under study are indistinguishable from one another. This enables a framework for system study where the infinite possibilities of system scenarios may be parsed to a workable, finite set. For most systems of interest, this finite set may still be too large for practical analysis, but the solution to this problem will be addressed later.

Total complexity is therefore the collection of every possible pairing of a scenario with the complexity response for a given system. This is, in fact, isomorphic with the

statistical complexity calculated by entropic methods. Yet, we reached the definition using two types of data: the energetic complexity values and the set of all possible scenarios.

With this new insight, it is clear that energetic complexity is not inconsistent with complexity as defined in the literature. Systems which consist of networks, have multiple levels of organization, anticipate the future, and have many "niches" (per Waldrop [67]) have more total complexity than systems that don't; systems with non-linear responses, disproportionate effects, and non-obvious cause-effect linkages (per Senge [59]) have more total complexity than systems without.

1.4.1 Complexity, Chaos, and Randomness.

As discussed, it is assumed complexity exists on a spectrum of behavior, that progresses from static (or linear) to complex to chaotic. A chaotic system is one whose state at time $t > t_0$ cannot be calculated beyond an arbitrary precision α. That is, an arbitrarily small error in initial conditions (at time t_0) can yield large errors in the calculation of the final state at time t. An energetically complex system may exhibit chaotic dynamics and therefore also be a chaotic system. The two concepts are not mutually exclusive.

Energetic complexity is a physical phenomenon, whereas chaos is a mathematical one. In this view, Ç serves as the backbone of the complexity spectrum of behavior, and can be used as a tool to determine the class of behavior. What is important is the relationship between Ç and highly chaotic systems.

One reason that entropy-based complexity metrics are so appealing is they approach zero for random processes as well as static ones. Most working in the complexity field accept this as a requirement for a good metric, although what happens between those limiting cases has little bearing. [44]

At first glance, it appears the Ç measure increases without bound as energy increases. However, recall that Ç is based on physical processes and is subject to physical limits; there is a limit to the amount of energy that physical systems may contain (e.g. particles in a box, stars, life forms). At the high entropy (disordered) limit, as energy of the particles increases (e.g. isochoric temperature increase), the particles accumulate so much kinetic energy that their collisions degrade their physical structure (e.g. molecular disruption). At this point, complexity falls. For a macroscopic "particle", as it degrades into smaller and less ordered components with the energy increase, the ability of this system to perform a function degrades. This degradation results in the activated energy transfers falling below the functional level. Completing the relationship, as energy is removed from a system, the kinetic energy of particles is reduced, and gradually drops below the functional threshold.

It should be noted that the exact relationship between complexity and entropy is system dependent and dependent on the entropy-changing process itself (which, per Li [44], is acceptable). For an isothermal volume-changing process at the high-entropy limit, as volume is increased, Ç decreases and approaches zero. Conversely, as volume is decreased, Ç increases until it reaches a point where the particles are contained so tightly that the kinetic energy of the particles falls below the functional (energy) threshold. Similar arguments can be made for other basic entropy-changing processes.

It is important to distinguish truly random processes from apparently random ones. A vehicle driving erratically might appear to exhibit random behavior to the outside observer; but what appears as random might in fact be very causal. What if the vehicle is under fire? Erratic motion (being unpredictable to the enemy), is in that case a highly effective strategy for survival, and is in fact a very complex behavior.

1.4.2 Agents and Energetic Complexity.

Agent-based models are of great importance to complexity theory, and measures of complexity should reflect increasing complexity with increasing activity of agents.

"Boids" (Bird-androids) are a type of agent based model using rule sets based on the motions of animal flocks, herds, and swarms. [15] Compare a 3-boid flock against 3 equivalent non-interacting aircraft. In going from point A to point B, which takes place over a particular epoch in both cases, non-interacting aircraft will make flight path adjustments due to atmospheric disturbances. A boid-aircraft will make these adjustments as well, but also make adjustments due to each of the boid guidance rules. Therefore the boid flock has higher energetic complexity than the set of non-interacting aircraft. Recalling the fact that non-cooperative flight is a subset of cooperative flight (equivalent to setting certain rule weightings to zero), if no boid rules are invoked, both systems have equivalent complexity.

Note also that human-piloted aircraft in formation interact with each other in similar fashion to the boid rules. Adjustments are made based on the relative positions and velocities of the other aircraft. Therefore, such a set of interacting aircraft can have the same (or greater) Ç than the boid flock.

1.4.3 Combat System Example: Tank.

Consider a tank system designed to strike a fixed target with four degrees of freedom (DoF): x, y, ϕ, and θ, where x and y are Cartesian position coordinates, ϕ is gun elevation, and θ is turret azimuth. A strike may be achieved by adjusting the state space $\mathbf{x}$ =$[x\ y\ \phi\ \theta]^T$ through a control vector $\mathbf{u}$.

First, consider the case where all degrees of freedom are enabled: $\mathbf{u}$= $[\frac{a}{t}\ \frac{b}{t}\ \frac{c}{t}\ \frac{d}{t}]^T$, where a, b, c, and d are constants (i.e. constant rate of motion from start to stop). If each DoF is exercised by the control vector, $Ç(f(\mathbf{x}, \mathbf{u})) = 8$. This sets a lower bound

on $Ç_A$.

In the next case, the same tank system is restricted to move in only 3 Degrees-of-Freedom (DoF), having lost the ability to rotate the turret due to damage or energy rationing. The vector $\mathbf{u} = [\frac{a}{t}\ \frac{b}{t}\ \frac{c}{t}\ 0]^T$ reflects the loss of control. If each available DoF is exercised by u, $Ç(f(\mathbf{x}, \mathbf{u})) = 6$, a lower bound for $Ç_A$ of this new system. This result is intuitive, since the loss of a degree of freedom should reflect a loss in the complexity of the system. The same trend is evident if we restrict the motion still further, such that $\mathbf{u} = [0\ 0\ \frac{c}{t}\ 0]^T$. In this case, Ç$= 2$.

Now consider different control policies, where the components of $\mathbf{u}$ do not drive the system at a constant rate. Suppose $c(t)$ is chosen to drive the tank gun elevation at a non-linear rate according to Figure 21(a). Here, $Ç = 2$ again. Has the metric failed to capture system complexity?

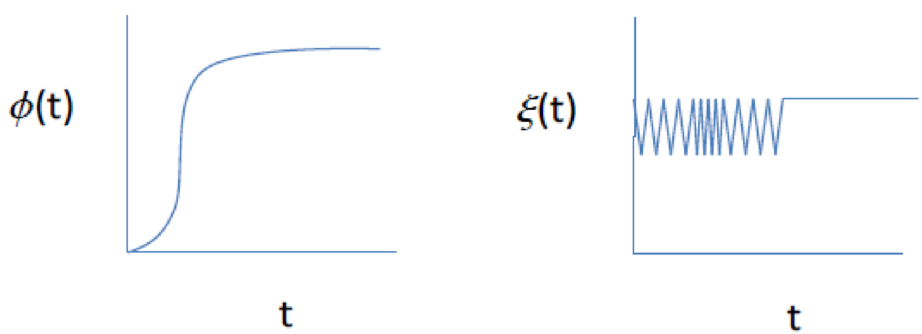

Figure 21. (a) Non-linear function of gun elevation, $\phi(t)$; (b) Motor brush position $\xi(t)$

This example illustrates why Ç is evaluated at a particular functional level; it prevents double-counting complexity. At the tank system functional level, this change in control function has no effect on Ç. To discover the effect, it is necessary to explore one functional level deeper: the gun elevation motor subsystem.

Suppose this is a standard Direct Current (DC) motor. Every time the brushes in the motor make or break contact (switching polarity and keeping the armature mov-

ing), kinetic energy goes through zero, generating Ç counts. Figure 21(b) illustrates this motion. The "missing" complexity exists at this lower subsystem.

Extending the tank example still further, suppose that the full-authority system is able to satisfy the range-to-target equation by adjusting the value of only one DoF. However, the controller opts to make movements in all 4 DoFs to reach the target. This results in an "excess" complexity, behavior which is not necessary to reach the goal.

This example demonstrates why it is important to keep in mind the goals and limitations of a complexity measure. Complexity is a reflection of a system's potential behaviors, not of efficient behaviors. The choice of control vector, given a system's complexity, is what controls efficiency. Irrational actors can implement control policies that produce highly complex behaviors, yet yield no benefit.

The effectiveness of a complex adaptive system can be measured by the choice of control vector against some objective function. That effectiveness, however, is not a measure of its complexity but what it does with that complexity.

In this tank example, total complexity may be calculated by determining a probability mass function for the scenario set and summing the product of that function with the complexity response for each scenario (as in Equation 50). Practically, the scenario set will consist of the most likely operating conditions. The constructed measure of total complexity may then be used to compare systems and determine suitability. Similarly, absolute complexity (as in Equation 48) provides additional insight into the complexity potential of the system, but total complexity gives a more complete picture for guiding system design.

1.5 Conclusions and Future Work

This research demonstrates the validity and use of a new measure of complexity in dynamic systems that may be more practically employed than prior measures. Energetic complexity can be used to compare systems in identical scenario sets, measure historical complexity, or set bounds on the absolute complexity.

As this is only an initial exploratory work, further validation is required. The behavior of this metric under scenarios of system collapse [**?**] or general evolution could yield insight into developing robust system architectures that are resistant to failure in varied and uncertain fitness landscapes.

Appendix B. Related work: Flight test results for UAVs using boid guidance algorithms

This appendix is a paper developed in the course of this research which was reviewed, accepted, and published in the proceedings of the 2012 Conference on Systems Engineering Research [16]. This paper documents flight test results using the boid guidance algorithms used in simulation to generate the research data.

2.1 Abstract

A critical technology for operating groups of Uninhabited Aerial Vehicles (UAVs) is distributed guidance. Distributed guidance allows an operator to command several vehicles at the same time, reduces operator workload, and adds redundancy to the system. Some of the leading software candidates for achieving distributed guidance are known as Boid Guidance Algorithms (BGAs), which are agent-based techniques relying on the interactions of simple behaviors.

Flight tests are crucial to the advancement of flight technologies such as BGAs, and this was identified as an important area for development. This paper presents the results from the 2005 flight tests of BGAs at NASA Dryden Flight Research Center with two RnR Products' APV-3 UAVs employing CloudCap Technology's Piccolo autopilot system.

Major challenges in these flight tests include the use of a waypoint-following system, limited computation resources, and management of safety procedures. The conclusions of this work include the need for using a path-following platform and completion of a full system optimization. This work is an important step in the development of a deployable distributed guidance system.

2.2 Introduction

The ability for a single operator to manage multiple aircraft is a sought-after goal. Possible benefits of this technology include improved operator efficiency, the opportunity for distributed function, redundancy in hardware, reduced production and operating costs, and improved safety. However, the management of several aircraft simultaneously is wrought with challenges.

This research stems from a collaborative project of NASA Dryden Flight Research Center (DFRC) and Ames Research Center (ARC) entitled Networked UAV Teams (NUAVT) [25]. The focus of the project was to show the feasibility of using cooperative UAVs to assist in forest fire fighting activities. However, the technologies developed are general enough for use in many different applications.

Previous approaches to this problem are computationally expensive and incur a significant scaling penalty with regard to group size [33] [34]. Since small UAVs typically have limited computing capacity, this problem is of high importance. BGAs have shown a great deal of success in coordinating large numbers of simulated aircraft [21] [68], but there are few known flight tests.

2.3 Boid Guidance Algorithms

Boid Guidance Algorithms (where "Boid" stands for "Bird android") are rule-based guidance methods inspired from observations of animal flocks and swarms [54]. It was proposed that these complex emergent behaviors could be explained if each animal agent were to follow a set of very simple rules as shown in Figure 22 [52]. The combination of these rules can lead to seemingly intelligent behavior.

The behavioral rules are combined by weighting the acceleration command of each behavior and summing. Determining the weightings is typically the most challenging aspect of working with BGAs.

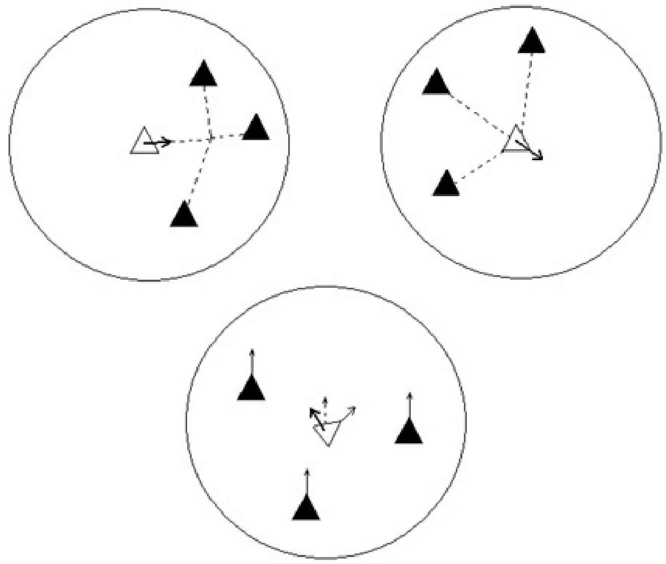

Figure 22. Clockwise from left: Flocking behavior, collision avoidance behavior, and heading matching behavior.

Previous work on BGAs involved exponentially scaling the behavior weightings [21]. That is, as one boid approached another, its collision avoidance behavior would get a higher weighting. The inverse would occur as the boid moved farther from its neighbors. This research did not use such a method as it incurs a greater on-line computation penalty yet presents similar optimization challenges. Instead, this implementation used a look-up table scheduled by a contingency management system.

BGAs were chosen for this work due to their low computation requirements, but were further modified so that most computation takes place off-line. This modification reduces the workload on the flight system. In addition, this system has a limited scaling penalty.

The rules used here are velocity matching (Equation 51), flock centering (Equation 52, where $\vec{x}_{center}$ is defined as in Equation 53), collision avoidance (Equation 54), target seeking (Equation 55), and obstacle avoidance (Equation 56), where $\vec{x}$, $\vec{v}$, and $\vec{a}_{(\cdot)}$ are the current boid position, velocity, and partial acceleration, respectively.

$$\vec{a}_{match} = \frac{\vec{v}_{nearest} - \vec{v}}{\| \vec{v}_{nearest} - \vec{v} \|} \tag{51}$$

$$\vec{a}_{flock} = \frac{\vec{x}_{center} - \vec{x}}{\| \vec{x}_{center} - \vec{x} \|} \tag{52}$$

$$\vec{x}_{center} = \frac{1}{n} \sum_{i=0}^{n} \vec{x}_i \tag{53}$$

$$\vec{a}_{col} = -\frac{\vec{x}_{nearest} - \vec{x}}{\| \vec{x}_{nearest} - \vec{x} \|} \tag{54}$$

$$\vec{a}_{seek} = \frac{\vec{x}_{waypoint} - \vec{x}}{\| \vec{x}_{waypoint} - \vec{x} \|} \tag{55}$$

$$\vec{a}_{obs} = -\frac{\vec{x}_{obstacle} - \vec{x}}{\| \vec{x}_{obstacle} - \vec{x} \|} \tag{56}$$

2.3.1 Contingency Management System.

Much like a linearized control system, using only one set of behavior weightings is valid for only a certain range of conditions. For a robust system, the weightings must change in response to changes in the environment.

To accomplish this, trigger flags were defined to indicate the state of the environment. For instance, if an aircraft passes within a specified range of its neighbor, the value of a flag is changed. Here, 48 different flag combinations were identified. For each combination, a set of weightings is specified.

For all behaviors except obstacle avoidance, nominal conditions are indicated by a "Level 2" flag. For instance, if the aircraft are nearby each other without being too close, headed in about the same direction, and have reached the target area, then

the flocking, collision avoidance, heading matching, and target seeking behavior flags will all be Level 2. The obstacle avoidance behavior flag has three levels: Level 3 for nominal, Level 2 if the aircraft is heading toward an obstacle, and Level 1 if a collision is imminent.

The parameters that trigger the flags (such as allowable distance between aircraft) must be determined by the user based upon aircraft performance characteristics. For an aircraft model with significant uncertainty, the parameters may be set arbitrarily restrictive at first, but eased as the aircraft model is refined.

2.3.2 Other modifications.

In addition to the contingency management system, other changes to the basic algorithm were required. Commands generated by the BGA were limited according to speed and turn magnitude, so as to reflect the limits of the control system. The turn limiting was based on a maximum load factor constrained by either the drag and minimum thrust or the stall limit [2].

To ensure the aircraft stayed within the prescribed boundaries for flight tests, when an aircraft came within a certain distance of the boundaries, it was given full power in the direction orthogonal to the boundary. The concern with this approach is that full priority is given to area containment at the expense of collision avoidance. However, the safety of the public and operators is paramount.

In addition to limiting the commands, some smaller modifications to the algorithm were required. Since the number of waypoints available to each aircraft was limited (see "Challenges" section), the commanded paths were modified to remove waypoints on relatively long and straight section. Further details on these modifications and the bases for calculations are found in Clark [15].

2.4 Optimization

The weightings for the BGA were optimized using a Simple Genetic Algorithm (SGA) [31]. Previous work showed success using SGAs for optimization of complex functions and BGAs in particular [26] [24].

SGAs use an evolutionary approach to optimize according to an objective function. Given an initial population of binary strings, an SGA will evaluate the fitness of each individual string. Then, using the processes of reproduction, crossover, and mutation, the SGA produces a new population of strings. With each successive generation, the probability for an improvement of overall fitness generally increases.

This research used a SGA with a population of 20, a generation gap of 0.9, a bit precision of 8, and a variable range of 0 to 100 for the five behavior weightings. The SGA ran for 500 generations after which the top performing individual was selected for inclusion in the BGA. The objective function consisted of a sum of the "Level 1" flags corresponding to each behavior, with an additional weight of 4 on the target seeking behavior flag. A Level 1 flag in either of the anti-collision behaviors resulted in an artificially large objective value of 5000, essentially taking it out of future populations.

Due to lack of computing resources for timely completion, only one set of behavior weightings was selected for optimization, with three others being set manually. The full contingency management system desired would include at least 48 such sets. Therefore, the obtained set was optimized for a specific set of conditions, and deviations from these nominal conditions can result in undesired performance results.

2.5 Challenges

There were several challenges to testing the BGA system, both self-imposed and exogenous. Perhaps the most serious impediment to achieving an ideal test of the BGAs was the use of an autopilot system employing waypoint-following logic. While

suitable for many applications, waypoint-following is generally inadequate for robust collision avoidance. Since collision avoidance is a key component of BGAs, the waypoint-following logic presented a major difficulty in ensuring test success.

In addition, the NASA ARC-developed interface program could accept only 15 waypoints per path for each aircraft, leading to larger distances between waypoints. Aircraft motion between the waypoints is not dictated by the BGA, thus larger waypoint separation results in an increased chance of collision.

A further constraint of the selected autopilot system was the requirement that the final waypoint either link back to the first point or be made an orbit point. To increase certainty in the aircraft positions, the final point in all paths was made an orbit point. The challenge is found in that the aircraft starts entering the orbit as soon as it crosses the next-to-last waypoint. This can lead to significant deviation from the path commanded by the BGA.

As mentioned earlier, the implemented BGA is minimally optimized. This constraint limited the tolerance of the algorithm to deviations from its optimized condition. It was therefore important to choose an optimization case that was a more challenging environment than expected in flight.

Some self-imposed safety features also presented challenges to performing a robust test of the algorithm. In order to ensure flight safety, the flight paths generated by the algorithm were inspected by the operators before the aircraft were allowed to follow them. This created two difficulties: the paths were not updated in flight, resulting in open-loop guidance; and accurate initial conditions were not available to the algorithm.

2.6 Flight test results

Flight tests were conducted by NASA DFRC and ARC in January 2005 at Edwards Air Force Base. The aircraft transited the test area from corner to corner, while avoiding virtual obstacles and one another. Figure 23 shows results from one such test, with diamonds and squares as the waypoints for the first and second aircraft, respectively.

In this scenario, the aircraft are travelling from lower left to upper right with no obstacles. The difficulty of establishing achievable initial conditions is visible as is the entry into orbit of the last waypoint. With the exception of the endpoints, the flight path meets the goal of coordinated group flight without collision or departure from the test area. In Figure 24, the selected aircraft initial conditions correspond well to the actual path entry points. However, due to the distance between the final orbit point and the next-to-last waypoint, there is significant departure from the desired path. This illustrates the need for the next-to-last waypoint to be near the final orbit point when using a waypoint-guidance system.

In Figure 25, the aircraft avoid simulated obstacles, although the path deviation of the northernmost aircraft nearly crosses one of the obstacle boundaries. Again, this is a result of the next-to-last waypoint being too far from the final orbit point. The aircraft safely navigate among four simulated obstacles in Figure 26. The distance between waypoints leads the aircraft to approach an obstacle closer than desired.

Figure 27 shows a clear violation of a simulated obstacle. This failure is due to the reduced number of available waypoints for the path. None of the waypoints are within the obstacle boundaries, so it was not an algorithm failure. The results shown in Figure 28 are very similar to those of Figure 27. The obstacle boundary violation is again due to the reduced number of available waypoints.

Figure 29 demonstrates a failure of the algorithm, as waypoints were placed within

the obstacle boundaries. This is due to the use of a minimal set of schedulable behavior weightings. If the full set were available, the contingency management system may have prevented the incursion. Also of note is the excursion of the aircraft with the dotted path early in the flight. This is another artifact of the waypoint-following algorithm used by the autopilot combined with inaccurate initial conditions selection.

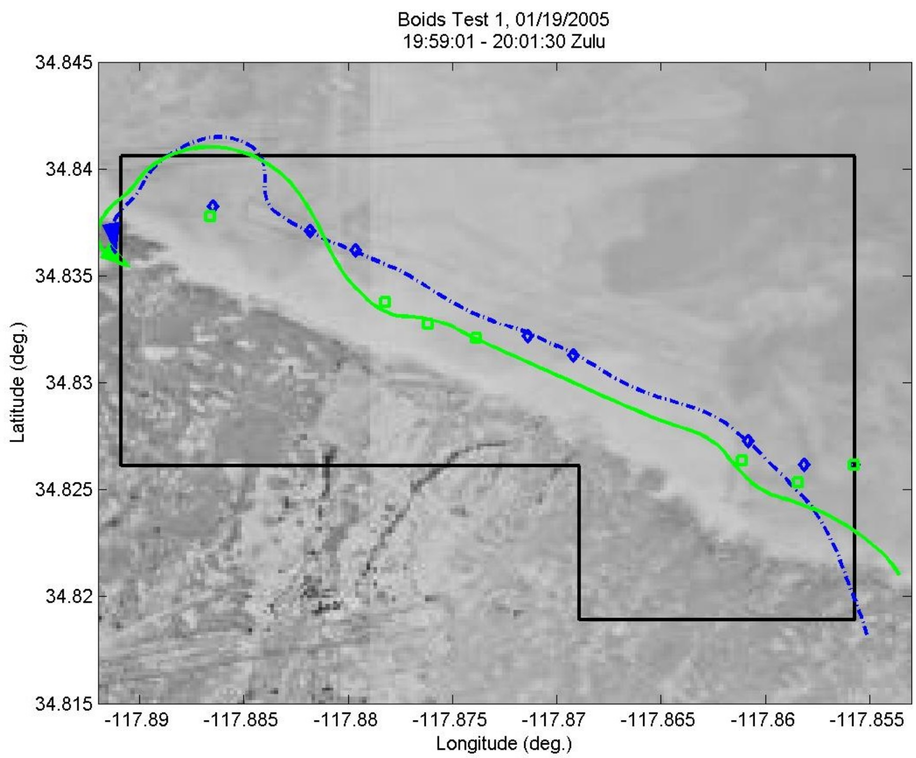

Figure 23. BGA Test Scenario 1

2.7 Conclusions and Future Work

Although more work is needed to develop a robust system, these flight tests have demonstrated an emerging capability for safely and efficiently UAVs utilizing boid algorithms. Future work should include a path-following system for robust collision avoidance and dynamic path development. On-aircraft computation and calculating a complete set of weightings would improve results. It would also be instructive to

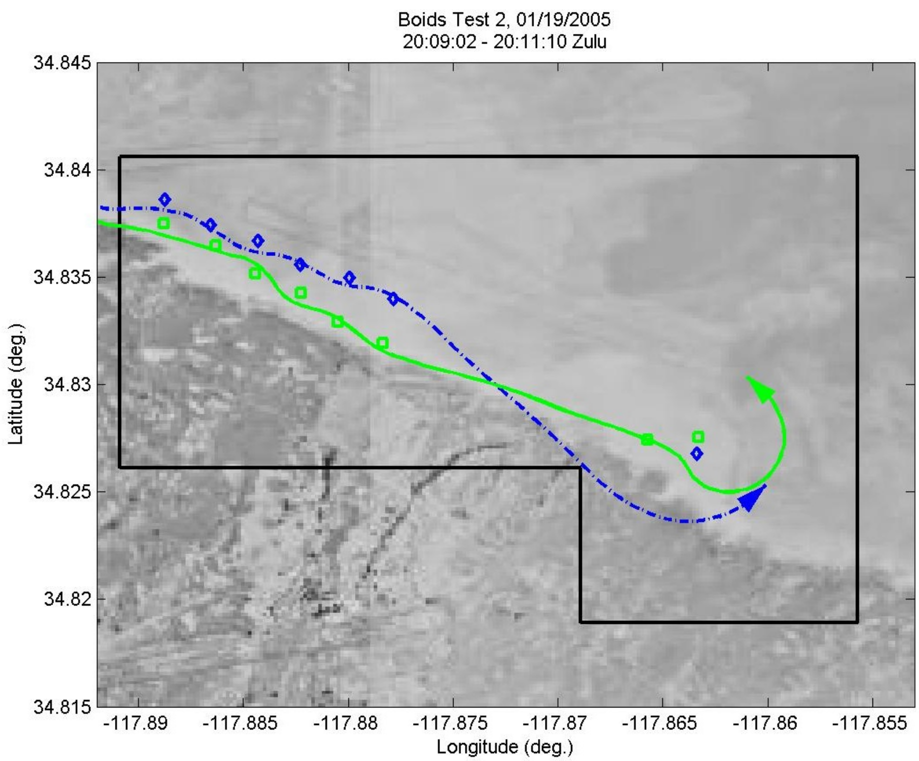

Figure 24. BGA Test Scenario 2

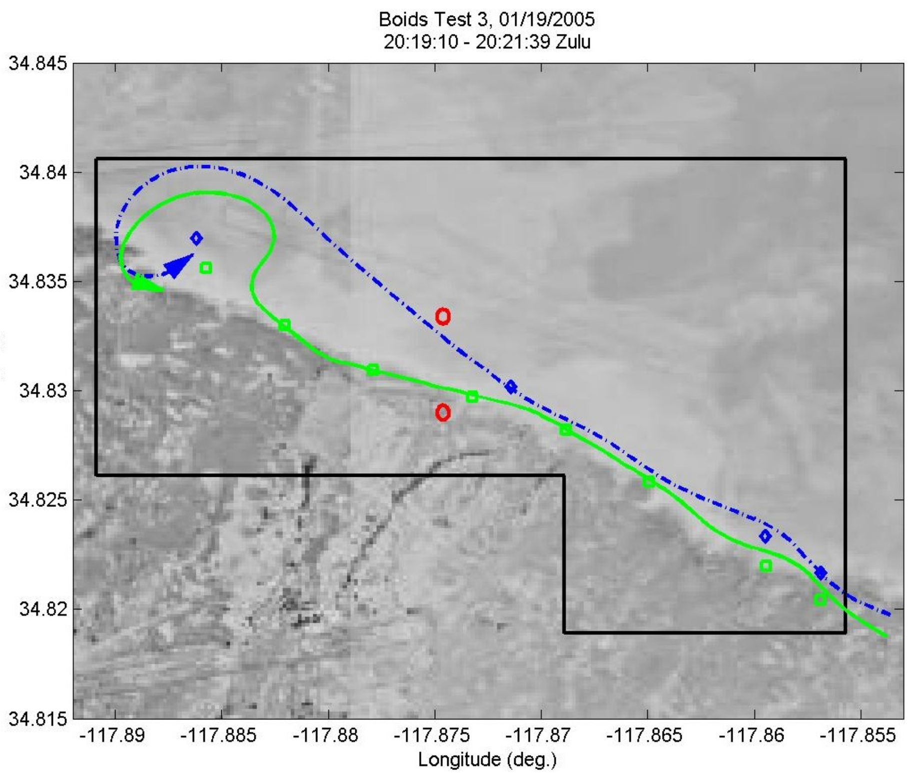

Figure 25. BGA Test Scenario 3

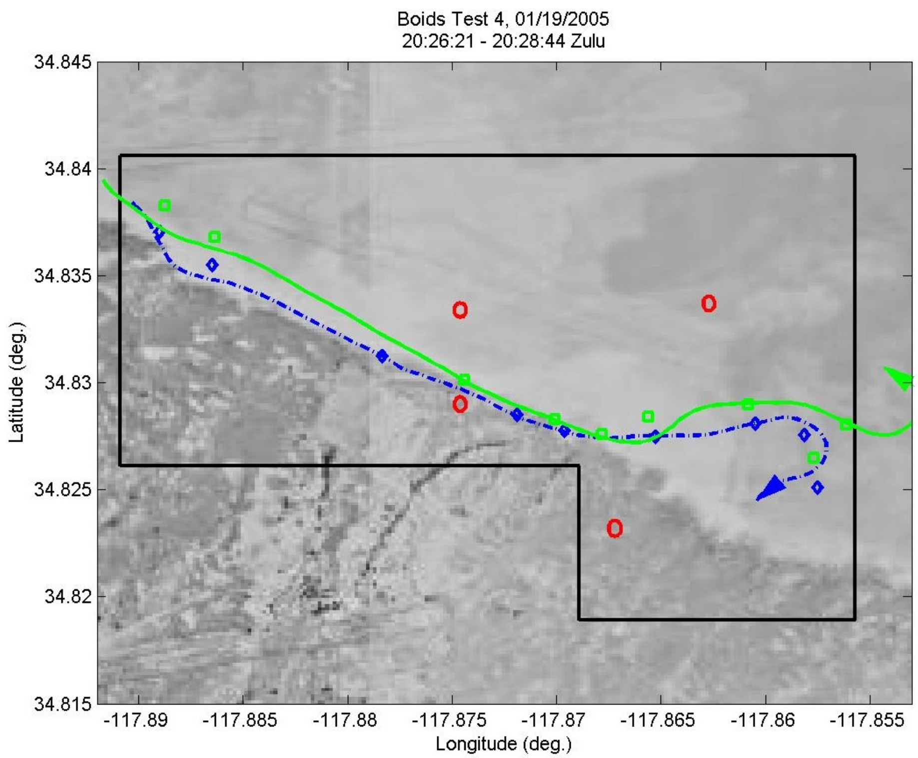

Figure 26. BGA Test Scenario 4

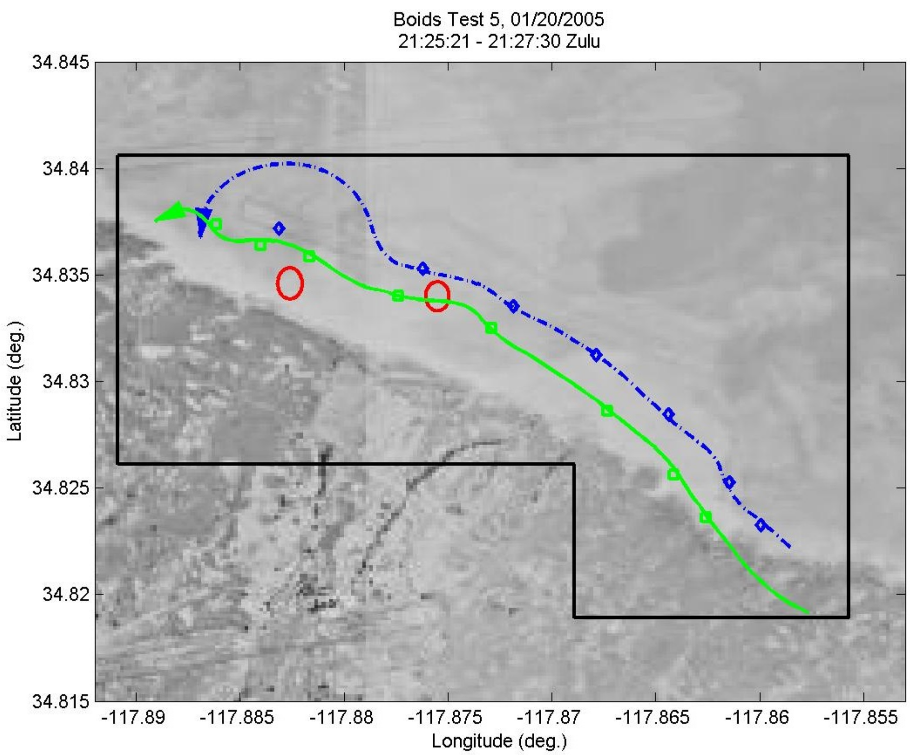

Figure 27. BGA Test Scenario 5

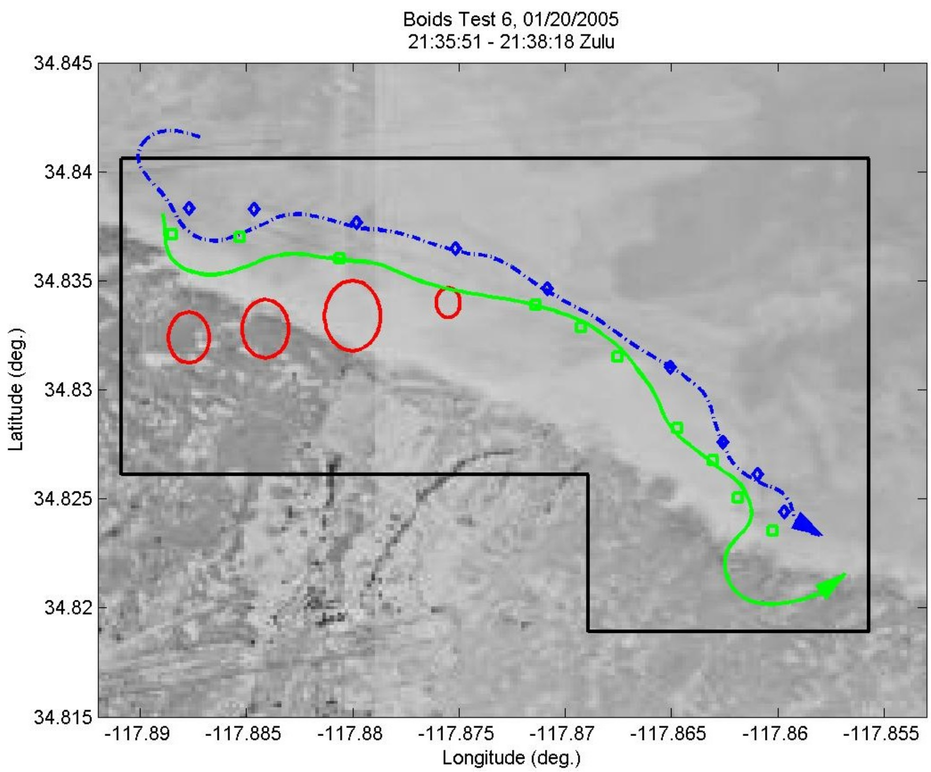

Figure 28. BGA Test Scenario 6

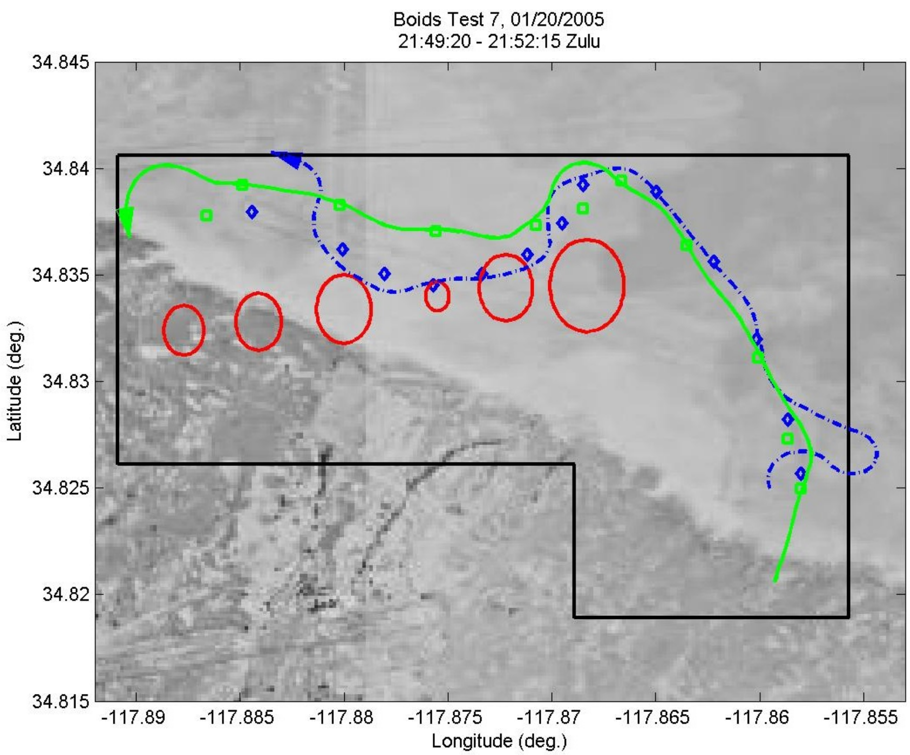

Figure 29. BGA Test Scenario 7

use “self-optimizing” techniques such as Particle Swarm Optimization [4] to improve adaptability. Tests exploring other emergent behaviors, such as orbiting or drag reduction [15], would be good next steps.

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14. ABSTRACT

While system functions, functionality, and complexity are widely used concepts in systems engineering, there is significant diversity in their definitions and no unified approach to measurement. This research establishes a method for measuring impacts to functionality in dynamic engineered systems based on changes in kinetic energy. This metric is applied at particular levels of abstraction and system scales, consistent with the established multiscale nature of systems. By measuring system behavior in context with expected scenarios, it is possible to estimate expected functionality or set bounds on a system's maximum functionality. Functionality and system effectiveness is heavily influenced by the amount of available energy and the information a system has about its environment. A framework is needed for quickly assessing the impact of changes in information in order to drive system architecture and design. This research relates functionality to the information content required to describe a system using principles from information theory and complexity theory.

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